Consider two single-turn co-planar, concentric coils of radii $R$ and $r$, with $R \gg r$, as shown in Figure. What is the mutual inductance between the two loops?

\[ \Phi_2 = B \cdot A = \frac{\mu_0 I_1}{2R} \pi r^2 \]

Thus magnetic flux inside the inner circle is:

The mutual inductance is then,

\[ M = \frac{N_2 \Phi_2}{I_1} = 1 \cdot \frac{\frac{\mu_0 I_1}{2R} \pi r^2}{I_1} = \frac{\mu_0 I_1 \pi r^2}{2R} \]
Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Compute the self-inductance of a solenoid with $N$ turns, length $L$, and radius $R$ with a current $I$ flowing through each turn.

Assuming field inside the solenoid uniform, we can calculate flux through its crosssection area as

$$\Phi = B \cdot A = \mu_0 \frac{N}{L} I \pi R^2$$

So the inductance is,

$$L = \frac{N \Phi}{I} = \frac{\mu_0 \pi N^2 R^2}{L}$$
Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: 
Student ID: 
Signature: 

Calculate the self-inductance of a toroid which consists of \( N \) turns and has a rectangular cross section, with inner radius \( a \), outer radius \( b \) and height \( h \), as shown in Figure.

Field inside a toroid:

\[ \oint \mathbf{B} \cdot d\mathbf{A} = \mu_0 I \text{ circ} \]

\[ \mathbf{B} \cdot d\mathbf{A} = \mu_0 NI \]

\[ B(r) = \frac{\mu_0 NI}{2\pi r} \]

Flux through the strip:

\[ d\Phi = \mathbf{B} \cdot d\mathbf{A} = \frac{\mu_0 NI h}{2\pi r} \]

Total flux:

\[ \Phi = \int d\Phi = \frac{\mu_0 NI h}{2\pi} \int_a^b \frac{b}{r} dr = \frac{\mu_0 NIh}{2\pi} \ln \left( \frac{b}{a} \right) \]

Inductance

\[ L = \frac{N^2 \Phi}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln \left( \frac{b}{a} \right) \]
A long solenoid with length l and a cross-sectional area A consists of \( N_1 \) turns of wire. An insulated coil of \( N_2 \) turns is wrapped around it, as shown in Figure. (i) Calculate the mutual inductance \( M \), assuming that all the flux from the solenoid passes through the outer coil. (ii) Relate the mutual inductance \( M \) to the self-inductances and of the solenoid and he coil.

\( (i) \quad M = \frac{N_2 \phi_{B2}}{I_1} \)

\[ \phi_{B2} = BA \quad \text{where} \quad B = \frac{\mu_0 N_1 I_1}{l} \]

\[ = \frac{\mu_0 N_1 I_1 A}{l} \]

\[ \Rightarrow M = \frac{\mu_0 N_1 N_2 I_1 A}{I_1 l} = \frac{\mu_0 N_1 N_2 A}{l} \]

\( (ii) \quad \) Since \( M = \frac{N_2 \phi_{B2}}{I_1} = \frac{N_1 \phi_{B1}}{I_2} \quad \Rightarrow \quad I_2 = \frac{N_1 \phi_{B1}}{M} \)

\[ \text{and} \quad I_1 = \frac{N_2 \phi_{B2}}{M} \]

\[ \Rightarrow \quad I_1 I_2 = \frac{M^2}{I_1 I_2} \]

\[ \Rightarrow \quad M = \sqrt{I_1 I_2} \]
A long solenoid with length $l$ and a radius $R$ consists of $N$ turns of wire. A current $I$ passes through the coil. Find the energy stored in the system.

$$U = \frac{1}{2} L I^2$$

$$L = \frac{N \Phi_B}{I}$$

$$\Phi_B = \vec{B} \cdot \vec{A} = BA = \left(\frac{\mu_0 NI}{l}\right) (\pi R^2)$$

$$\Rightarrow L = \frac{\mu_0 N^2 R^2 \pi}{I}$$

$$\Rightarrow U = \frac{1}{2} \frac{\mu_0 N^2 I^2 R^2}{l}$$