A 20.0 \mu F capacitor is charged to a potential difference of 800 V. The terminals of the charged capacitor are then connected to those of an uncharged 10.0 \mu F capacitor. Compute,

a) the original charge of the system,
b) the final potential difference across each capacitor,
c) the final energy of the system, and
d) the decrease in energy when the capacitors are connected.

\[ Q_{\text{initial}} = \frac{V \cdot C}{\text{initial}} = \frac{800 \text{ V} \cdot 20.0 \mu F}{10.0 \mu F} = 16 \times 10^3 \times 10^{-6} C = 1.6 \times 10^{-3} C \]

\[ Q_1 = 2Q_2 < \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_1}{20.0 \mu F} = \frac{Q_2}{10.0 \mu F} \]

\[ 3Q_2 = Q_{\text{initial}} \]

\[ Q_2 = \frac{16}{3} \times 10^{-3} C \]

\[ V_1 = V_2 = \left( \frac{16 \times 10^{-3} C}{10.0 \mu F} \right) = \frac{16 \times 10^{-2}}{3} \text{ V} \]

\[ E_{\text{final}} = \frac{1}{2} \left( 20.0 \mu F + 10.0 \mu F \right) \left( \frac{16}{3} \times 10^{-2} \text{ V} \right)^2 = 15.0 \times \frac{16^2}{3^2} \times 10^{-2} \text{ J} = \frac{64}{15} \text{ J} \]
d) \[ \Delta E = E_{\text{final}} - E_{\text{initial}} \]

\[ E_{\text{initial}} = \frac{1}{2} (200 \text{ mF})(200 \text{ V})^2 = \frac{20 \times 6.4 \times 10^4 \times 10^{-6}}{2} \]

\[ = 6.4 \times 10^{-2} \text{ J} = \frac{3.2}{5} \text{ J} \]

\[ E_{\text{final}} = 15 \times \left( \frac{15}{5} \right)^2 \times 10^{-2} \text{ J} \]

\[ \Delta E = \frac{6.4}{15} \text{ J} - \frac{3.2}{5} \text{ J} = -\frac{3.2}{15} \text{ J} \]

\[ \Delta E = E_f - E_i = \frac{6.4}{15} \text{ J} - \frac{3.2}{5} \text{ J} = -\frac{3.2}{15} \text{ J} \]

Decrease in energy is \[ \frac{32}{15} \text{ J} \]
A parallel plate capacitor is made from two plates 12.0 cm on each side and 4.50 mm apart. Half of the space between these plates contains only air, but the other half is filled with plexiglas of dielectric constant 3.40 as shown in the figure below. An 18.0 V battery is connected across the plates.

a) What is the capacitance of this combination?
   (Hint: Can you think of this capacitor as equivalent to two capacitors in parallel?)

b) How much energy is stored in the capacitor?

c) If we remove the plexiglas but change nothing else, how much energy will be stored in the capacitor?

$$A_{\text{plate}} = (12.0 \text{ cm})^2 = 144 \text{ cm}^2$$
$$C = \frac{\varepsilon A}{d}$$

$$C_{\text{eq}} = C_1 + C_2$$
$$C_1 = \frac{\varepsilon_0 A_{\text{plate}}/2}{d}$$
$$C_2 = 3.40 \varepsilon_0 \frac{A_{\text{plate}}/2}{d}$$

$$C_{\text{eq}} = 2.20 \varepsilon_0 \frac{A_{\text{plate}}}{d} = \frac{2.20 \times (\varepsilon_0 \times 10^{-12} \text{ F/m})}{4.50 \text{ mm}} = 6.23 \times 10^{-11} \text{ F}$$

$$E_1 = \frac{1}{2} C_{\text{eq}} (18.0 \text{ V})^2 = \frac{1}{2} \frac{2.20 \varepsilon_0 A_{\text{plate}}}{d} (18.0 \text{ V})^2 = 9.14 \times 10^{-8}$$

$$E_2 = \frac{1}{2} C_1 (18.0 \text{ V})^2 = \frac{1}{2} \frac{\varepsilon_0 A_{\text{plate}}}{d} (18.0 \text{ V})^2 = 4.59 \times 10^{-9}$$
Two long, coaxial cylindrical conductors are separated by vacuum as shown in the figure below. The inner cylinder has radius $r_a$ and linear charge density $+\lambda$. The outer cylinder has inner radius $r_b$ and linear charge density $-\lambda$. Find the capacitance per unit length for this capacitor.

\[
\frac{2\pi r L}{E} = \frac{\lambda L}{\varepsilon_0}
\]

\[
E = \frac{\lambda}{2\pi r \varepsilon_0}
\]

\[
\Phi_{\text{Total}} = \Phi = V \cdot C
\]

Capacitor formula

\[
V_a = -\int_{r_b}^{r_a} \frac{\lambda}{2\pi \varepsilon_0} \, dr = \frac{\lambda}{2\pi \varepsilon_0} \ln \left| \frac{r_b}{r_a} \right|
\]

\[
Q = \lambda L
\]

\[
\frac{C}{L} = \frac{Q}{V_{ab} \cdot L} = \frac{2\pi \varepsilon_0}{\ln \left| \frac{r_b}{r_a} \right|}
\]

Per unit length

\[
C = \frac{Q}{V_{ab}} = \frac{2\pi \varepsilon_0 L}{\ln \left| \frac{r_b}{r_a} \right|}
\]

Total capacitance