Consider two particles. One particle is of mass $M = 10^{-26}$ kg and charge $Q = 3 \times 10^{-19}$ C and the other is of charge $q = 10^{-19}$ C and mass $m = 2 \times 10^{-30}$ kg. Assume the heavy particle is stationary. Consider a one dimensional motion such that initially the light particle is directly moving away from the heavy one at a distance $r = 10^{-10}$ m and has speed $v = 10^6$ m/s.

(i) Show that the gravitational interaction can be neglected.

(ii) Calculate the speed of the light particle when it is escaped from the electrical influence of the heavy particle. (In other words when the separation between the particles is infinite.)

Take $k = 1/4\pi\varepsilon_0 = 10^9$ Nm$^2$/C$^2$ and $G = 7 \times 10^{-11}$ Nm$^2$/kg$^2$.

\[ F_G = \frac{G M m}{r^2} = 7 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \frac{(10^{-26} \text{ kg})(2 \times 10^{-30} \text{ kg})}{(1 \times 10^{-10} \text{ m})^2} \]

\[ F_G = 11 \times 10^{-6} \text{ N} \]

\[ F_E = k \frac{Q q}{r^2} = 10^{10} \text{ Nm}^2/\text{C}^2 \frac{(3 \times 10^{-19} \text{ C})(10^{-19} \text{ C})}{(10^{-10} \text{ m})^2} = 3 \times 10^{-8} \text{ N} \]

Since $F_E \gg F_G$ gravitational interaction can be neglected.

\[ i) \quad \frac{F_E}{F_G} = \frac{10^{10} N}{11 \times 10^{-6} N} \approx 10^{16} \]

\[ \frac{F_E}{F_G} \approx \infty \]

\[ \text{(Since } F_E \gg F_G \text{ gravitational interaction can be neglected.)} \]

\[ ii) \quad \text{At infinity } V \to 0, \text{ use energy conservation for light particle} \]

\[ U_{Ei} + K_i = U_{Ef} + K_f \]

\[ K Q q + \frac{1}{2} m v_i^2 = 0 + \frac{1}{2} m v_f^2 \]

\[ 10^{10} \text{ Nm}^2/\text{C}^2 \frac{(3 \times 10^{-19} \text{ C})(10^{-19} \text{ C})}{(1 \times 10^{-10} \text{ m})^2} + \frac{1}{2} \left(2 \times 10^{-30} \text{ kg}\right)(10^6 \text{ m/s})^2 = \frac{1}{2} \left(2 \times 10^{-30} \text{ kg}\right) v_f^2 \]

\[ v_f \approx 3 \times 10^6 \text{ m/s} \]
Consider a chain of three charges. The one on the left end of the chain has charge $q_1 = -e$; the middle one has charge $q_2 = +e$ and the one at the right end has charge $q_3 = +e$. Assume the spatial separation between the charges are the same and denoted by “$\alpha$”. Assume the chain is created by a sequence of two processes. First process: the middle charge is brought from very far away (infinity) next to the left charge. Second process: the right charge is brought from very far away next to the other two charges.

(i) Calculate the (minimum) work done by the external force in both processes.

(ii) Calculate the (minimum) external work required to destroy this charge configuration by separating the charges to infinite distances.

\[
W = U_1 - U_2 = \int F_e \cdot d\tau = \frac{e^2}{4\pi \varepsilon_0} \alpha
\]

1\textsuperscript{st} \quad W_1 = - \frac{e^2}{4\pi \varepsilon_0} \int_0^{\alpha} \frac{dr}{r^2} = \frac{e^2}{4\pi \varepsilon_0 \alpha}

2\textsuperscript{nd} \quad W_2 = \frac{e^2}{4\pi \varepsilon_0} \int_0^{2\alpha} \frac{dr}{r^2} = - \frac{e^2}{8\pi \varepsilon_0 \alpha}

\quad \quad i) \quad W_T = -(W_1 + W_2) = - \left( \frac{e^2}{4\pi \varepsilon_0 \alpha} - \frac{e^2}{8\pi \varepsilon_0 \alpha} \right) = - \frac{e^2}{8\pi \varepsilon_0 \alpha}
Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name:                      Student ID:                      Signature:
Calculate the electric potential of a uniformly charged disk of radius R and total charge Q along its symmetry axis.

\[ V = \frac{Q}{2\pi \varepsilon_0} \int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}} = \frac{Q}{2\pi \varepsilon_0} \left[ \sqrt{x^2+y^2} \right]_0^R \]

\[ V = \frac{Q}{2\pi \varepsilon_0} \left( \sqrt{x^2+r^2} - x \right). \]
Closed book. No calculators are to be used for this quiz. Quiz duration: 10 minutes

Name: ___________________________  Student ID: ___________________________  Signature: ___________________________

Consider two nested (coaxial) cylindrical conductors of height $H$ and radii $r$, $R$ ($R > r$) respectively. Assume the height $H$ is much longer than the radii so that the cylinders can be considered infinitely long and the fringe effects can be neglected. A positive charge $+Q$ is uniformly distributed on the outer surface of the inner cylinder, while the inner surface of the outer cylinder is uniformly charged to $-Q$. Calculate the potential difference between the two cylinders.

\[
V = -\int \mathbf{E} \cdot d\mathbf{r}^2
\]

\[
= -\int_0^R \mathbf{E} \cdot d\mathbf{r}^2 - \int_R^{r'} \mathbf{E} \cdot d\mathbf{r}^2
\]

Electric field is zero outside.

\[
V = -\int_0^R \mathbf{E} \cdot d\mathbf{r}^2 - \int_R^{r'} \frac{Q}{2\pi \varepsilon_0 r'} dr
\]

\[
V = -\frac{Q}{2\pi \varepsilon_0} \ln \left(\frac{r}{r'}\right)
\]

\[
V = \frac{Q}{2\pi \varepsilon_0} \ln \left(\frac{R}{r}\right)
\]