Efficiency Wage

- A risk neutral agent working for a firm
- Assume two effort and output levels
- There is limited liability: \( w_0, w_1 \geq 0 \)
- We have already studied this problem
- If \( (p_1 - p_0)(s_1 - s_0) \geq \psi \)
  - Firm prefers to induce high effort and the solution is
    \[
    w_0 = 0, \quad w_1 = \frac{\psi}{p_1 - p_0} > 0
    \]
  - Positive wage is called efficiency wage because it induces the agent to exert high (efficient level of) effort
- For further development of this idea see
  - Shapiro and Stiglitz (1984)

Financial Contracts

- A risk averse entrepreneur has a project that needs \( I \) dollars of investment
- Return on the project is \( s_1 \) or \( s_0 \)
- Entrepreneur’s effort is either 0 or 1
- \( p_i \) is probability of \( s_1 \) if effort is \( i = 0, 1 \)
- A risk neutral lender offers a contract that specifies repayment levels conditional on return: \( (r_0, r_1) \)

Let \( w_0 = s_0 - r_0 \) and \( w_1 = s_1 - r_1 \)

The problem is identical to what we have studied before

At the solution
\[
\begin{align*}
r_1 &= s_1 - h \left( \psi + \psi \frac{1 - p_1}{p_1 - p_0} \right) < s_1 \\
r_0 &= s_0 - h \left( \psi + \psi \frac{p_1}{p_1 - p_0} \right) > s_0
\end{align*}
\]
Financial Contracts

- Lender’s expected payoff is
  \[ p_1 s_1 + (1 - p_1) s_0 - C - I \]
  
  where
  \[ C = p_1 h \left( \psi + \psi \frac{1 - p_1}{p_1 - p_0} \right) + (1 - p_1) h \left( \psi - \psi \frac{p_1}{p_1 - p_0} \right) \]
  
  is the cost (to the lender) of moral hazard

- The project will be funded only if
  \[ I \leq L = p_1 s_1 + (1 - p_1) s_0 - C \]

- Under complete information project will be funded as long as
  \[ I \leq T = p_1 s_1 + (1 - p_1) s_0 \]

- Moral hazard creates credit rationing for projects with investment requirement in \([L, T]\)

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CARA-Normal Model

- Commonly used model
  - agent’s preferences have constant absolute risk aversion (CARA)
  - outcome is normally distributed
  - contracts are linear

- More precisely the agent’s preferences can be represented by
  \[ u(w, a) = -e^{-\eta(w - ca)^2/2} \]
  
  where \( \eta > 0 \) is the coefficient of absolute risk aversion (\( \eta = -u''/u' \)).

- Outcome \( q \) is equal to effort \( a \) plus noise \( \epsilon \)
  \[ q = a + \epsilon \]

  where \( \epsilon \) is normally distributed with zero mean and variance \( \sigma^2 \)

- Contracts are limited to be linear
  \[ w = t + sq \]

  where \( t \) is the fixed and \( s \) is the performance related component of the payment

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CARA-Normal Model

- Principal’s problem is
  \[ \begin{aligned}
  & \max_{a, t, s} \mathbb{E}[q - w] \\
  \text{subject to} & \quad \mathbb{E}[-e^{-\eta(w - ca^2)^2}] \geq u(\mathbb{E}) \\
  & \quad a \in \arg\max_{\hat{a}} \mathbb{E}[-e^{-\eta(w - \hat{a}^2)^2}] 
  \end{aligned} \]

  where \( \mathbb{E} \) is the expectation operator and \( u(\mathbb{E}) \) is the reservation utility of the agent

- Note that
  \[ w - \frac{ca^2}{2} = t + sq - \frac{ca^2}{2} = t + s(a + \epsilon) - \frac{ca^2}{2} \]
CARA-Normal Model

Therefore, agent’s utility can be written as
\[ E[-e^{-\eta(t+s(a+\epsilon)-\frac{c^2}{2})}] = -e^{-\eta(t+s\epsilon-\frac{c^2}{2})}E[e^{-\eta s\epsilon}] \]

We can show that
\[ E[e^{\eta s\epsilon}] = \eta^2 s^2 \sigma^2 / 2 \]

CARA-Normal Model

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Agent’s maximization problem is
\[ \max_a \left( t + sa - \frac{c^2}{2} - \eta^2 s^2 \sigma^2 / 2 \right) \]

with solution \( a = \frac{s}{c} \)

CARA-Normal Model

- Density of \( N(\mu, \sigma^2) \)

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left( \frac{(x-\mu)^2}{2\sigma^2} \right) dx \]

Therefore, for any \( \gamma \)
\[ E[\exp(\gamma x)] = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(\gamma x) \exp \left( \frac{x^2}{2\sigma^2} \right) dx \]
\[ = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left( \frac{(x-\gamma^2 \sigma^2)^2 - \gamma^2 (\sigma^2)^2}{2\sigma^2} \right) dx \]
\[ = \exp \left( \frac{\gamma^2 \sigma^2}{2} \right) \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left( \frac{(x-\gamma^2 \sigma^2)^2}{2\sigma^2} \right) dx \]
\[ = \exp \left( \frac{\gamma^2 \sigma^2}{2} \right) \]

Principal’s problem becomes
\[ \max_{t, s} \frac{s}{c} - (t + \frac{s^2}{c}) \]
subject to
\[ t + \frac{s^2}{2c} - \eta s^2 \sigma^2 / 2 = w \]
which is solved as
\[ s = \frac{1}{1 + \eta c \sigma^2} \]
Properties of the Solution

\[
a = s/c, \quad s = \frac{1}{1 + \eta \sigma^2}
\]

- As the agent becomes more risk-averse, performance sensitive component \( s \) decreases \( \eta \uparrow \rightarrow s \downarrow \)
- As risk increases, performance sensitive component and effort decrease \( \sigma^2 \uparrow \rightarrow s, a \downarrow \)
  - \( s \) exposes the agent to risk
  - Principal has to compensate agent for increased risk
  - Better to reduce \( s \)
- As cost of effort for the agent decreases, performance sensitive component and effort increases \( c \downarrow \rightarrow s, a \uparrow \)

Inefficient Behavioral Responses to Incentive Contracts

- Performance sensitive contracts have the cost of exposing the agent to risk
- They may also have unintended consequences
- Agents may act in a way that hurts employers
- Many jobs are complex and have aspects that are difficult to specify in contracts
- Explicit contracts may induce the agent to focus on aspects included in the contract to the detriment of others

Inefficient Behavioral Responses to Incentive Contracts

- Football quarterback Ken O’Brien mid-80s
  - Used to throw a lot of interceptions
  - Received a contract that penalized every time he threw the ball to opposition
  - It worked: He threw fewer interceptions
  - But largely because he refused to throw the ball at all
- AT&T programmer contracts
  - Rewarded on the number of lines of code
  - Resulted in longer programs than was necessary
- Examples of multi-tasking
  - Holmstrom and Milgrom (1991)
  - Prendergast (1999)
  - Survey with simple models and empirical evidence

A Simple Model

- Agent is risk neutral
  \[ u(w, a) = w - \frac{ca^2}{2} \]
- Outcome \( q \) is equal to effort \( a \) plus noise \( \varepsilon \)
  \[ q = a + \varepsilon \]
  where \( \varepsilon \sim N(0, \sigma^2) \)
- Compensation is based on
  \[ \tilde{q} = \mu a + \varepsilon \]
- Agent privately knows \( \mu \), principal believes \( \mu \sim N(1, \sigma^2_\mu) \)
- Models the divergence between the privately and socially optimal effort level
  - \( \sigma^2_\mu = 0 \Rightarrow \) no divergence
- Example: If agent is rewarded on quantity produced, then she may work hard even if she knows there is no demand for it
A Simple Model

- Contracts are limited to be linear
  \[ w = t + s\tilde{q} \]
- A model with both hidden information and hidden action
- We assume that principal cannot design screening contracts
- See Baker (1992)

Agent’s problem is to choose \( a \) to maximize
\[
t + s\tilde{q} - \frac{ca^2}{2} = t + s(\mu a + \varepsilon) - \frac{ca^2}{2}
\]
Solved as
\[
a = \frac{\mu s}{c}
\]
Principal’s problem is
\[
\max_{t,s} E[q - w] \\
\text{subject to} \\
E[w - \frac{ca^2}{2}] \geq \overline{w}
\]
Substituting for \( a \) and using the fact that the constraint must hold as equality, problem becomes
\[
\max_s \frac{s}{c} - \frac{s^2}{2c(\sigma^2\mu + 1)}
\]
This is solved as
\[
s = \frac{1}{\sigma^2\mu + 1}
\]
Optimal contract in the standard model with a risk-neutral agent is
\( s = 1 \)
Here, incentives are muted \((s < 1)\) in order to constrain inefficient behavioral responses
See Prendergast (1999) for empirical evidence