Regulation

- Analyzed in two contexts:
  - **Procurement**: The firm supplies a good to the government
    - Defense contracts
  - **Regulation**: The firm supplies a good to consumers on behalf of government
    - Utilities (electricity, gas), telecommunications, railroads

- There are many natural monopolies in utilities, telecommunication, infrastructure, and transportation industries
  - Due to large fixed costs, average cost is decreasing even at large production levels
  - It may be socially efficient to have a monopoly

- Government needs to regulate to prevent them acting as a monopolist
  - But it doesn’t know the relevant characteristics (e.g., technology) and/or does not observe actions (e.g., cost reducing effort) of the firm

Typically, government uses accounting (cost) data and reimburses a fraction of firm’s monetary expenditures $c$

Its aim is to obtain the highest societal welfare in providing the good

We will model it as follows: government pays $c$ and a net transfer $t$ to the firm

$$t = a - bc$$

- $a$: a fixed fee
- $b$: fraction of costs born by the firm (measures the power of incentives)

Possible contracts

- **cost-plus (cost-of-service)**: $b = 0$
  - firm does not bear any cost
  - low powered: firm does not have an incentive to lower cost
- **fixed-price (price-cap)**: $b = 1$
  - government does not reimburse any cost; pays only a fixed fee
  - high powered: firm is the residual claimant for its cost savings
- **incentive (cost-sharing)**: $0 < b < 1$

Procurement: 1960 US

- 40.9% cost-plus
- 13.6% incentive
- 31.4% fixed-price

Regulation:

- historically: cost-of-service
- more recently: price-cap and incentive regulation
Regulation

- Government: provide incentives to reduce cost and extract firm’s rent
- Fixed-price
  - Incentives: Good
    - Firm is the residual claimant
    - Provides efficient effort
  - Rent extraction: Bad
    - Rent is non-negative when cost is high
    - Exogenous reduction in cost benefits the firm
- Cost-plus
  - Incentives: Bad
  - Rent-extraction: Good
- Optimal contract
  - Perfect information about technology: fixed-price
  - Imperfect information: incentive (cost-sharing) contract

Early models


Perfect Information

- Regulator knows $\theta$ (therefore, can deduce $e = \theta - c$)
- Its problem is:
  $$\min_{(e,t)} \{\theta - e + t\} \text{ s.t. } t - e^2/2 \geq 0, \quad e \geq 0$$
- The constraint must bind: $t = e^2/2$
  $$\min_{e>0} \{\theta - e + e^2/2\}$$
- If the solution is in the interior ($e^* > 0$)
  $$-1 + e^* = 0 \Rightarrow e^* = 1$$
- Comparing with the boundary ($e = 0$) shows that the solution is in the interior
Perfect Information

- Effort is efficient
  \[ \text{marginal cost} = \psi'(e^*) = e^* = 1 = \text{marginal benefit} \]
  and
  \[ t^* = 0.5 \quad \text{firm receives no rent} \]
- Regulator pays the firm a fixed total payment \( P = \theta - 0.5 \)
- Induces the firm to minimize \( c + e^2/2 \)
- This is a price-cap (fixed-fee) regulation
  \[ t = a - (c - e^*) \]
  where
  \[ a = \psi(e^*) = 0.5 \]
  \[ e^* = \theta - e^* = \theta - 1 \]

Unobservable \( \theta \) and \( e \)

- What happens if regulator offers first-best?
  \[ c_L = \theta_L - 1, c_H = \theta_H - 1 \text{ and } t_L = t_H = 0.5 \]
- For the efficient type \( (\theta_L) \)
  \[ \begin{align*}
    & \text{Effort required to obtain cost } c_L \text{ is } 1 \\
    & \text{Effort required to obtain cost } c_H (e') \text{ is } 1 - \Delta \theta \leq 1:
      \[ c_H = \theta_H - 1 = \theta_L - e' \]
  \end{align*} \]
- Since transfers are the same, she prefers to mimic the inefficient type
  - Usual recipe for optimal contract:
    \[ \begin{align*}
      & \text{Efficient type } (\theta_L): \\
      & \quad \ast \text{ provides first best effort} \\
      & \quad \ast \text{ obtains positive rent} \\
      & \text{Inefficient type } (\theta_L): \\
      & \quad \ast \text{ provides less than first best effort (to reduce efficient type’s rent)} \\
      & \quad \ast \text{ obtains no rent}
    \end{align*} \]

Solving the Problem

1. \( IR_L \) does not bind:
   \[ t_L - e_L^2/2 \geq t_H - (e_H - \Delta \theta)^2/2 > t_H - e_H^2/2 \geq 0 \]
2. \( IC_L \) binds, otherwise
   \[ t_L - e_L^2/2 > t_H - (e_H - \Delta \theta)^2/2 > t_H - e_H^2/2 \geq 0 \]
   and the regulator can decrease \( t_L \)
3. \( IR_H \) binds, otherwise (by (1)) can decrease \( t_L \) and \( t_H \) by the same small amount
4. \( e_H \leq e_L + \Delta \theta \) or \( c_H \geq c_L \): \( IC_L \) and \( IC_H \) imply
   \[ e_H^2/2 - (e_L + \Delta \theta)^2/2 \leq t_H - t_L \leq (e_H - \Delta \theta)^2/2 - e_L^2/2 \]
   - Result follows from simple algebra
5. \( IC_H \) can be ignored: follows from steps (2) and (4)
Solving the Problem

Therefore, the problem is equivalent to

\[
\min_{(t_i, e_i)} \quad p(t_L + \theta_L - e_L) + (1 - p)(t_H + \theta_H - e_H)
\]
subject to
\[
\begin{align*}
t_H - e_H^2/2 &= 0 \\
t_L - e_L^2/2 &= t_H - (e_H - \Delta \theta)^2/2
\end{align*}
\]

Note that

\[
t_L - e_L^2/2 = \Delta \theta(e_H - \Delta \theta/2)
\]
\[
e_H \geq \Delta \theta \implies \text{type } \theta_L \text{ obtains a rent}
\]

Solving the Problem

Substituting in the constraints, the problem is equivalent to

\[
\begin{align*}
\min_{e_L,e_H \geq 0} \quad & \left\{ p(e_H^2/2 + e_L^2/2 - (e_H - \Delta \theta)^2/2 - e_L) + (1 - p)(e_H^2/2 - e_H) \right\} \\
\text{or} \\
\max_{e_L,e_H \geq 0} \quad & -\left\{ p(e_H^2/2 + e_L^2/2 - (e_H - \Delta \theta)^2/2 - e_L) + (1 - p)(e_H^2/2 - e_H) \right\}
\end{align*}
\]

The objective function is strictly concave and the constraint functions \(e_L\) and \(e_H\) are concave.

Therefore, if a solution exists Kuhn-Tucker conditions will uniquely identify it.

The Lagrangean is given by

\[
\mathcal{L} = -\left\{ p(e_H^2/2 + e_L^2/2 - (e_H - \Delta \theta)^2/2 - e_L) + (1 - p)(e_H^2/2 - e_H) \right\} + \lambda_L e_L + \lambda_H e_H
\]

Solving the Problem

Critical points are given by

\[
\begin{align*}
-p(e_L - 1) + \lambda_L &= 0 \\
-(p\Delta \theta + (1 - p)(e_H - 1)) + \lambda_H &= 0
\end{align*}
\]
\[
\lambda_L, \lambda_H, e_L, e_H \geq 0, \quad \lambda_L e_L = \lambda_H e_H = 0
\]

First equation and \(\lambda_L \geq 0 \implies e_L > 0 \implies \lambda_L = 0\), which implies \(e_L = 1\).

Similarly the assumption \(\Delta \theta \leq 1 - p\) implies that
\[
e_H = 1 - \frac{p}{1 - p} \Delta \theta
\]

and \(e_H \geq \Delta \theta\), as required.
Properties of the Solution

- Familiar “no distortion at the top” and “under-provision for inefficient types” result
- Optimal regulation: offer a menu so that
  - Efficient firm chooses a price-cap regulation
    - A constant payment \( P_L = t_L + \theta_L - e_L \)
    - \( t_L = a_L - b_L c \), \( a_L = t_L + \theta_L - e_L \) and \( b_L = 1 \)
  - Inefficient firm chooses a cost-sharing arrangement
    - \( t_H = a_H - b_H c \), \( a_H = t_H + \theta_H (\theta_H - e_H) \) and \( b_H = e_H = 0.5 \)

Financial Contracts and Credit Rationing

- Why are certain worthy projects not funded?
- If there is excess demand of funds, the interest rate should increase to equate demand with supply
- The reason might be adverse selection
  - As the interest rate increases only riskier projects apply
- Most popular article:

A Simple Model

- There is a population (unit mass) of risk-neutral borrowers
- Each borrower owns a project that requires an investment of 1
- Two types of borrowers: risky \( (\theta_r) \) and safe \( (\theta_s) \)
  - \( \text{prob}(\theta_s) = q \)
- Each project has an uncertain return \( X \in \{0, R_i\} \), \( R_i \) with prob. \( p_i \)
  - \( \theta_r = (p_r, R_r) \) and \( \theta_s = (p_s, R_s) \)
- Assume \( p_s > p_r \) and \( R_r > R_s \)
- We will look into two scenarios:
  1. \( p_i R_i = m > 1 \): Both projects worth investing
  2. \( p_r R_r < 1 < p_s R_s \): Only safe project worth investing
- Single bank with total amount of funds \( \max\{q, 1 - q\} < \alpha < 1 \)
- Net expected payoff of the bank if it lends to type \( i \) and requires a repayment of \( D_i \) is given by \( p_i D_i - 1 \)
  - Assumes limited liability: if the return is zero, the borrower defaults and bank gets zero repayment

Symmetric Information

- The problem of the bank facing borrower of type \( i \) is
  \[
  \max_{D_i \geq 0} \quad p_i D_i - 1 \\
  \text{subject to} \\
  p_i (R_i - D_i) \geq 0
  \]
- For now assume \( p_i R_i = m > 1 \)
- Solution is simple: lend all \( \alpha \) and set \( D_i = R_i \)
- Expected payoff of the bank is \( \alpha (p_i R_i - 1) = \alpha (m - 1) > 0 \)
AdverseSelection

- Assume that a bank contract is simply $D$
- $D > R_s \Rightarrow$ only risky apply $\Rightarrow$ best to set $D = R_r$
  
  payoff $= (1-q)(p_r R_r - 1) = (1-q)(m-1)$
- $D \leq R_s \Rightarrow$ both apply $\Rightarrow$ lend all $\alpha$ at $D = R_s$
  
  payoff $= \alpha[q(p_s R_s - 1) + (1-q)(p_r R_s - 1)] = \alpha[qm + (1-q)p_r R_s - 1]$

- If $(1-q)(m-1) < \alpha[qm + (1-q)p_r R_s - 1]$
  
  $\Rightarrow D = R_s$
  
  $\Rightarrow$ some borrowers cannot get credit
  
  $\Rightarrow$ credit rationing

Second Best

- A contract is $\{(x_s, D_s), (x_r, D_r)\}$
  
  $x_i$ is probability of getting a loan
- The bank’s problem is
  
  $\max_{(x_i,D_i)} \quad qx_s(p_s D_s - 1) + (1-q)x_r(p_r D_r - 1)$

  subject to
  
  $x_i p_i(R_i - D_i) \geq 0, \quad i = s, r$

  $x_i p_i(R_i - D_i) \geq x_j p_j(R_j - D_j), \quad i, j = s, r$

  $0 \leq x_i \leq 1, \quad i = s, r$

  $qx_s + (1-q)x_r \leq \alpha$

- If bank offers the first best menu
  
  $\{(x_s, D_s), (x_r, D_r)\} = \{(\alpha, R_s), (\alpha, R_r)\}$

  Risky type mimics the safe type

Solving the Problem

- This suggests the reduced problem:

  $\max_{(x_i,D_i)} \quad qx_s(p_s D_s - 1) + (1-q)x_r(p_r D_r - 1)$

  subject to

  $x_i p_i(R_i - D_i) \geq 0, \quad i = s, r$

  $x_r p_r(R_r - D_r) \geq x_s p_s(R_s - D_s)$

  $x_r \geq x_s$

  $0 \leq x_i \leq 1, \quad i = s, r$

  $qx_s + (1-q)x_r \leq \alpha$

- This should be familiar by now

  $\Rightarrow$ IR constraint of the high type $\theta_r$ does not bind

  $\Rightarrow$ IC constraint of the low type $\theta_s$ does not bind

  $\Rightarrow$ Monotonicity in the allocation $x_r \geq x_s$

- The original problem (P) is equivalent to the reduced problem (RP)
1. Constraints of (P) imply those of (RP): From the two IC constraints:

\[ x_r R_s - x_s R_s \leq x_r D_r - x_s D_s \leq x_r R_r - x_s R_r \]

which implies \( x_r \geq x_s \)

2. At the solution to (RP) IC constraint holds as equality: otherwise can increase \( D_r \)
   - Here we use \( x_r > 0 \). If \( x_r = 0 \), then by Step (1) \( x_s = 0 \), which cannot be a solution to (RP)

3. Solution of (RP) satisfy the constraints of (P)
   - IC and IR constraints of (RP) imply IR for type \( \theta_r \)
   - Step (2) and \( x_r \geq x_s \) imply IC for type \( \theta_s \)

\[ p_s [x_s (R_s - D_s) - x_r (R_s - D_r)] = p_s [x_s R_s - x_s R_s + x_r R_r - x_s R_r] \]
\[ = p_s (x_r - x_s) (R_r - R_s) \geq 0 \]

Suppose \( x_s > 0 \). Then, \( D_s = R_s \)

Substitute the IC and IR into the objective function

\[ \max_{x_s, x_r} [q(p_s R_s - 1) - (1 - q)p_r (R_r - R_s)] + (1 - q)x_r (p_r R_r - 1) \]

Since we assumed \( p_i R_i = m > 1 \)

\[ x_r = 1 > \alpha \]

\( x_s > 0 \) implies

\[ q(p_s R_s - 1) - (1 - q)p_r (R_r - R_s) \geq 0 \]

Together with monotonicity and budget constraint

\[ x_s = \frac{\alpha - (1 - q)}{q} < \alpha \]

IC for risky type \( (R_r - D_r = x_s (R_r - D_s)) \) and \( D_s = R_s \) imply

\[ D_r = x_s R_s + (1 - x_s) R_r > R_s = D_s \]

If \( q(p_s R_s - 1) - (1 - q)p_r (R_r - R_s) < 0 \)

\[ x_s = 0 \]

Interpretation

- The bank trades off the rents extracted from risky borrowers with ability to lend all its funds
- When safe borrowers are numerous enough, wants to lend to them, which leads to rent for risky borrowers
- But rather than financing everybody by setting \( D = R_s \), the bank gives risky borrowers preferential access by setting \( x_r > x_s \) in exchange for higher repayment
- Credit rationing disappears: The only ones not fully funded are safe borrowers, but they are indifferent about being funded
- Credit rationing reappears if we instead assume \( p_r R_r < 1 < p_s R_s \)
  - Bank wants to turn away risky borrowers (i.e., set \( x_r = 0 \)), but cannot do it without also denying the safe ones (since \( x_r \geq x_s \))
  - Either everybody is funded (if \( (q p_s + (1 - q) p_r) R_s \geq 1 \)) or nobody, i.e., there is a financial collapse