Instructions

- Please write your name in the space provided at the top.
- Answer all questions.
- Write your answers in the space provided for each answer.
- Show enough of your work so that your reasoning can be followed.
- You may detach the last two pages and use as scrap paper.
- Time allowed: 90 minutes.

<table>
<thead>
<tr>
<th>Question</th>
<th>Max</th>
<th>You get</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>30 pts</td>
<td></td>
</tr>
<tr>
<td>Question 2</td>
<td>30 pts</td>
<td></td>
</tr>
<tr>
<td>Question 3</td>
<td>40 pts</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100 pts</td>
<td></td>
</tr>
</tbody>
</table>

Good Luck!
1. (30pts.) Consider the following extensive form game:

(a) (10pts.) Assume that $a = 1$ and find the set of pure strategy Nash equilibria and subgame perfect equilibria.

(b) (10pts.) Find the range of $a$ for which $S$ is the unique subgame perfect equilibrium outcome.

(c) (10pts.) Find the range of $a$ for which $(C, c)$ is the unique Nash equilibrium outcome.
You may continue your answer to Question 1 on this page
You may continue your answer to Question 1 on this page
2. (30pts.) A seller ($S$) can make a costly investment to make her product more valuable for the buyer ($B$). Assume that if the seller invests $x$, then her cost is $x^2$ and the value of the buyer is $v + x$, where $v > 0$.

(a) (10pts.) Assume first that the game has three stages:

- **Stage I** Seller chooses investment $x \geq 0$
- **Stage II** Buyer observes $x$ and offers a price $p \geq 0$
- **Stage III** Seller observes $p$ and either accepts ($a$) or rejects ($r$) the offer

If the seller rejects the offer, then payoff of the buyer is zero whereas the payoff of the seller is $-x^2$. If she accepts the offer, then the payoff of the buyer is $v + x - p$ and the payoff of the seller is $p - x^2$. Find the subgame perfect equilibria of this game.

(b) (10pts.) Consider the same scenario as in part (a) except that it is the seller who makes a price offer and the buyer who responds. Find the subgame perfect equilibria of this game.

(c) (10pts.) Consider the same game but now suppose that the buyer can commit to give a share $\alpha \in [0, 1]$ of his value to the seller before the seller makes the investment decision. More precisely, the game has two stages:

- **Stage I** Buyer chooses share $\alpha \in [0, 1]$
- **Stage II** Seller observes $\alpha$ and chooses investment $x \geq 0$

Payoff of the buyer is $(1 - \alpha)(v + x)$ and the payoff of the seller is $\alpha(v + x) - x^2$. Find the subgame perfect equilibrium of this game.
You may continue your answer to Question 2 on this page
3. (40pts.) Consider the infinitely repeated version of the following game

\[
\begin{array}{c|cc}
\text{Player 2} & C & D \\
\hline
\text{Player 1} & 4, 4 & 0, 6 \\
 & 6, 0 & 1, 1 \\
\end{array}
\]

The payoff of player \(i\) to any infinite sequence of payoffs \(\{u_{it}\}\) is given by the normalized discounted sum of payoffs

\[(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} u_{it},\]

where \(\delta \in (0, 1)\). For what values of \(\delta\), if any, the following strategies constitute subgame perfect equilibria?

(a) (20pts.) Choose \(C\) in period 1. Choose \(C\) after any history in which the outcome in the last period is \((C, C)\). Choose \(D\) after any other history.

(b) (20pts.) Choose \(C\) in period 1 and then do whatever your opponent did last period.
You may continue your answer to Question 3 on this page
You may continue your answer to Question 3 on this page
You may use as scrap paper
You may use as scrap paper