Instructions

- Please write your name in the space provided at the top.
- Answer all questions.
- Write your answers in the space provided for each answer.
- Show enough of your work so that your reasoning can be followed.
- You may detach the last two pages and use as scrap paper.
- Time allowed: 75 minutes.

<table>
<thead>
<tr>
<th>Question</th>
<th>Max</th>
<th>Your Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>30 pts</td>
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<tr>
<td>Question 2</td>
<td>30 pts</td>
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<tr>
<td>Question 3</td>
<td>40 pts</td>
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<td>Total</td>
<td>100 pts</td>
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Good Luck!
1. (30pts.) Consider the following extensive form game:

(a) (10pts.) Assume that $x = 1$ and find the set of pure strategy Nash equilibria and subgame perfect equilibria.

(b) (10pts.) Find the range of $x$ for which $(R, u)$ is the unique subgame perfect equilibrium outcome.

(c) (10pts.) Find the range of $x$ for which $L$ is a Nash equilibrium outcome.
You may continue your answer to Question 1 on this page
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There is a project for which player 1 can exert effort $e \geq 0$ that costs her $c(e) = 0.5e^2$. If player 1 and player 2 can come to an agreement, then a total value of $v(e) = e$ is produced, which can be allocated between the two players. Effort also produces a value $y(e) = ke$, where $k \in [0, 1]$, that player 1 can obtain for herself if player 1 and 2 fail to agree. The game has three stages:

**Stage I** Player 1 chooses effort $e \geq 0$

**Stage II** Player 2 observes $e$ and chooses an offer $\alpha \in [0, 1]$

**Stage III** Player 1 observes $\alpha$ and either agrees ($a$) or rejects ($r$) the offer.

If player 1 accepts the offer, then her payoff is $\alpha e - 0.5e^2$ and player 2’s payoff is $(1 - \alpha)e$. If she rejects the offer, then player 1’s payoff is $ke - 0.5e^2$ and player 2’s payoff is zero. The following figure represents the game.

(a) **(20pts.)** What is the subgame perfect equilibrium effort choice?

(b) **(10pts.)** Suppose that player 2 can choose $k \in [0, 1]$ before the above game is played. What would he choose? What would be the equilibrium effort?
You may continue your answer to Question 2 on this page
3. (40pts.) Consider the infinitely repeated version of the following game

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
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<tbody>
<tr>
<td>Player 1</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>1,1</td>
</tr>
<tr>
<td>D</td>
<td>0,3</td>
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</table>

The payoff of player $i$ to any infinite sequence of payoffs $\{u_{it}\}$ is given by the normalized discounted sum of payoffs

$$
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} u_{it},
$$

where $\delta \in (0, 1)$.

(a) **(20pts.)** Consider the following strategy:

• Choose $D$ in period 1.
• Choose $D$ after any history in which both players have always played $D$.
• Choose $H$ after any other history.

For what values of $\delta$, if any, both players playing this strategy constitutes a subgame perfect equilibrium?

(b) **(20pts.)** Suppose that the game is given by the following bimatrix

<table>
<thead>
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<tr>
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<td>H</td>
</tr>
<tr>
<td>H</td>
<td>0,0</td>
</tr>
<tr>
<td>D</td>
<td>1,3</td>
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For what values of $\delta$, if any, both players playing the strategy in part (a) constitutes a subgame perfect equilibrium?
You may continue your answer to Question 3 on this page
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