ECON 333/MGEC 333/LAW 435
GAME THEORY AND STRATEGY
Midterm Examination II

Instructions

• Please write your name in the space provided at the top.
• Answer all questions.
• Write your answers in the space provided for each answer.
• Show enough of your work so that your reasoning can be followed.
• You may detach the last two pages and use as scrap paper.
• Time allowed: 75 minutes.

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<tr>
<th>Question</th>
<th>Max</th>
<th>Your Grade</th>
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<tr>
<td>Question 1</td>
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<td>Question 2</td>
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<td>Question 3</td>
<td>40 pts</td>
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<td>Total</td>
<td>100 pts</td>
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Good Luck!
1. **(30pts.)** Consider the following extensive form game:

![Game Tree]

(a) **(10pts.)** Assume that $x = 2$ and find the set of pure strategy Nash equilibria and subgame perfect equilibria. Is there any Nash equilibrium outcome that is not subgame perfect? If the answer is yes, explain why it is not subgame perfect.

(b) **(10pts.)** For what range of $x$ the unique Nash equilibrium outcome is $(R, l)$?

(c) **(10pts.)** For what range of $x$ there are two subgame perfect equilibrium outcomes?
You may continue your answer to Question 1 on this page
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2. (30pts.) The manager of a firm (\(M\)) makes a wage offer \(w\) to an employee (\(E\)). After this, the employee decides whether to exert high effort \(h\) or low effort \(l\). After observing the effort choice of the employee, the manager decides whether to rehire (\(r\)) the employee or not (\(n\)). If she does not rehire, then the players receive their payoffs and the game ends. If she rehires, then in the next period they receive the same payoffs as this period. Both the manager and the employee discounts the future by \(\delta \in (0, 1)\).

High effort produces \(v\) for the firm but costs \(c\) to the employee whereas low effort produces and costs nothing. Also assume that wage offer can be any real number in the interval \([0, v]\). The game can be represented as in the following figure, where the first payoff number is the manager’s:

\[
\begin{align*}
&\text{M} \quad E \\
&\quad w \\
&\quad M \quad M \\
&h \quad l \\
&r \quad n \quad r \quad n \\
&\quad (1 + \delta)(v - w), (1 + \delta)(w - c) \quad v - w, w - c \quad -(1 + \delta)w, (1 + \delta)w \quad -w, w
\end{align*}
\]

(a) (15pts.) Assume that \(v, c,\) and \(\delta\) can be any real numbers such that \(v > c > 0\) and \(\delta \in (0, 1)\). Under what conditions on these parameters, is there a subgame perfect equilibrium outcome in which the employee exerts high effort and the manager rehires the worker? Fully specify such an equilibrium.

(b) (15pts.) Let \(v = 2, c = 1, \delta = 1/2\) and find the subgame perfect equilibrium outcomes of this game.
You may continue your answer to Question 2 on this page
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3. **(40pts.)** Consider the infinitely repeated version of the following game

<table>
<thead>
<tr>
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<th>Player 2</th>
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<tbody>
<tr>
<td>Player 1</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>2, 2</td>
</tr>
<tr>
<td>L</td>
<td>3, 1</td>
</tr>
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The payoff of player $i$ to any infinite sequence of payoffs $\{u_{it}\}$ is given by the normalized discounted sum

$$(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} u_{it},$$

where $\delta \in (0, 1)$.

(a) **(20pts.)** For what values of $\delta$, if any, the following strategy profile constitutes a subgame perfect equilibrium?

- Play $H$ in period 1.
- Play $H$ after any history in which both players have always played $H$.
- Play $L$ after any other history.

(b) **(20pts.)** For what values of $\delta$, if any, the following strategy profile constitutes a subgame perfect equilibrium?

- Play $H$ in period 1.
- Play $H$ if $(H, H)$ or $(L, L)$ has been played in the last period.
- Play $L$ if $(H, L)$ or $(L, H)$ has been played in the last period.
You may continue your answer to Question 3 on this page
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You may use as scrap paper
You may use as scrap paper