ECON 333/MGEC 333/LAW 435  
GAME THEORY AND STRATEGY  
Midterm Examination I

Instructions

• Please write your name in the space provided at the top.
• Answer all questions.
• Write your answers in the space provided for each answer.
• Show enough of your work so that your reasoning can be followed.
• You may detach the last two pages and use as scrap paper.
• Time allowed: 75 minutes.

<table>
<thead>
<tr>
<th>Max</th>
<th>You get</th>
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<tbody>
<tr>
<td>Question 1</td>
<td>40 pts</td>
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<tr>
<td>Question 2</td>
<td>30 pts</td>
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<tr>
<td>Question 3</td>
<td>30 pts</td>
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<td><strong>Total</strong></td>
<td><strong>100 pts</strong></td>
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Good Luck!
1. (40pts.) Consider the following game:

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<tbody>
<tr>
<td>T</td>
<td>4.2</td>
<td>3.0</td>
<td>1.1</td>
</tr>
<tr>
<td>M</td>
<td>1.2</td>
<td>2.4</td>
<td>0.3</td>
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<tr>
<td>B</td>
<td>1.1</td>
<td>4.2</td>
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(a) (20 pts) Find the set of pure strategy Nash equilibria of this game.

(b) (20 pts) Is there any other (mixed strategy) Nash equilibrium of this game? (If your answer is yes, you must find all of them. If no, you must prove that there is no other Nash equilibrium)
You may continue your answer to Question 1 on this page
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2. (30 pts) Suppose that two firms are competing in a market and their products are imperfect substitutes for one another. The demand functions for the products of firm 1 and 2 are given respectively as

\[
q_1(p_1, p_2) = \begin{cases} 
  a - bp_1 + p_2, & \text{if } a - bp_1 + p_2 \geq 0 \\
  0, & \text{if } a - bp_1 + p_2 < 0
\end{cases}
\]

\[
q_2(p_1, p_2) = \begin{cases} 
  a - bp_2 + p_1, & \text{if } a - bp_2 + p_1 \geq 0 \\
  0, & \text{if } a - bp_2 + p_1 < 0
\end{cases}
\]

where \(p_1\) and \(p_2\) are prices charged by firm 1 and firm 2, respectively. The firms can charge any non-negative price, and we assume that \(a > 0\) and \(b > 1\). For simplicity assume that the costs of each firm are equal to zero, and each firm aims to maximize its own profits. In other words, firm \(i = 1, 2\) aims to maximize \(p_i q_i(p_1, p_2)\).

(a) (10 pts) Formulate this situation as a strategic form game.

(b) (20 pts) Find the set of all pure strategy Nash equilibria.
You may continue your answer to Question 2 on this page
You may continue your answer to Question 2 on this page
3. (30 pts) Player 1 and 2 are trying to share a cake of size 100. They each simultaneously submit a real number in the interval [0, 100]. If the sum of their numbers is smaller than or equal to 100, then each receives the number she submitted. If the sum is greater than 100, then the player who submitted the smaller number, say $x$, receives that number while the other receives $100 - x$. If the sum is greater than 100 and they are equal, then each receives 50.

(a) (10 pts) Formulate this situation as a strategic form game.
(b) (10 pts) Show that (50, 50) is a Nash equilibrium
(c) (10 pts) Is there any other Nash equilibrium? Prove your answer.
You may continue your answer to Question 3 on this page
You may continue your answer to Question 3 on this page
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You may use as scrap paper