Levent Koçkesen

Instructions

- Please write your name in the space provided at the top.
- Answer all questions.
- Write your answers in the space provided for each answer.
- Show enough of your work so that your reasoning can be followed.
- You may detach the last two pages and use as scrap paper.
- Time allowed: 90 minutes.

<table>
<thead>
<tr>
<th>Question</th>
<th>Max</th>
<th>You get</th>
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<tbody>
<tr>
<td>Question 1</td>
<td>40 pts</td>
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<tr>
<td>Question 2</td>
<td>30 pts</td>
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<tr>
<td>Question 3</td>
<td>30 pts</td>
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<td>Total</td>
<td>100 pts</td>
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Good Luck!
1. (40pts.) Consider the following game:

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<thead>
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<th>L</th>
<th>C</th>
<th>R</th>
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<tbody>
<tr>
<td>T</td>
<td>3.1</td>
<td>0.0</td>
<td>1, −1</td>
</tr>
<tr>
<td>M</td>
<td>0.0</td>
<td>3.1</td>
<td>2.0</td>
</tr>
<tr>
<td>B</td>
<td>1.2</td>
<td>1.4</td>
<td>4.3</td>
</tr>
</tbody>
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(a) (20 pts) Find the set of pure strategy Nash equilibria of this game.

(b) (20 pts) Is there any other (mixed strategy) Nash equilibrium of this game? (If your answer is yes, you must find all of them. If no, you must prove that there is no other Nash equilibrium)
You may continue your answer to Question 1 on this page
You may continue your answer to Question 1 on this page
Two students simultaneously decide how much time to spend on a joint project. If the times spent are $t_1$ and $t_2$, then the value of the project to each player is $t_1 + t_2 - t_1 t_2$. The cost of time $t_i$ to player $i$ is $\frac{1}{2} t_i^2$, $i = 1, 2$, and the payoff of a player is the value of the project minus the cost of effort. Each player’s time is a real number between zero and one, i.e., $t_i \in [0, 1]$.

(a) (10 pts) Formulate this situation as a strategic form game.

(b) (10 pts) Find the players’ best response correspondences.

(c) (10 pts) Find the set of Nash equilibria of this game.
You may continue your answer to Question 2 on this page
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3. (30 pts) Two firms 1 and 2 simultaneously choose time to spend on research $x_1 \geq 0$ and $x_2 \geq 0$. Suppose that firms’ payoff functions are given by

\[
\begin{align*}
u_1(x_1, x_2) &= \begin{cases} 
10 - x_1, & \text{if } x_1 \geq x_2 \\
-x_1, & \text{if } x_1 < x_2
\end{cases} \\
u_2(x_1, x_2) &= \begin{cases} 
10 - x_2, & \text{if } x_2 \geq x_1 \\
-x_2, & \text{if } x_2 < x_1
\end{cases}
\end{align*}
\]

(a) (15pts) Show that $x_1 = x_2 = 5$ is a Nash equilibrium.

(b) (15pts) Find the set of all Nash equilibria. (You may find necessary and sufficient conditions or calculate best response correspondences.)
You may continue your answer to Question 3 on this page
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You may use as scrap paper
You may use as scrap paper