1. (a) **Tit-For-Tat**: The behavior of a player who adopts this strategy depends only on the last period’s outcome. Therefore, we can group all the histories into those with the last period’s outcome being \((C, C)\), \((C, D)\), \((D, C)\), or \((D, D)\). We have to test optimality of the strategy after all such histories using the one-shot-deviation property. We will check optimality from the perspective of player 1, which is without any loss of generality since the game is symmetric.

   i. **Histories that end with \((C, C)\)**

   Playing according to the strategy yields:
   
   \[
   (C, C) \quad (C, C) \quad (C, C) \quad (C, C) \quad \cdots \\
   2 \quad 2 \quad 2 \quad 2 \quad \cdots
   \]

   whose normalized discounted sum is 2.

   One-shot deviation (OSD) yields
   
   \[
   (D, C) \quad (C, D) \quad (D, C) \quad (C, D) \quad \cdots \\
   3 \quad 0 \quad 3 \quad 0 \quad \cdots
   \]

   whose normalized discounted sum is \(3/(1 + \delta)\). Optimality requires

   \[
   2 \geq \frac{3}{1 + \delta}
   \]

   or

   \[
   \delta \geq \frac{1}{2}
   \]

   ii. **Histories that end with \((C, D)\)**

   Playing according to the strategy yields:
   
   \[
   (D, C) \quad (C, D) \quad (D, C) \quad (C, D) \quad \cdots \\
   3 \quad 0 \quad 3 \quad 0 \quad \cdots
   \]

   whose normalized discounted sum is \(3/(1 + \delta)\).

   One-shot deviation (OSD) yields
   
   \[
   (C, C) \quad (C, C) \quad (C, C) \quad (C, C) \quad \cdots \\
   2 \quad 2 \quad 2 \quad 2 \quad \cdots
   \]

   whose normalized discounted sum is 2. Optimality requires

   \[
   \delta \leq \frac{1}{2}
   \]

   iii. **Histories that end with \((D, C)\)**

   Playing according to the strategy yields:
   
   \[
   (C, D) \quad (D, C) \quad (C, D) \quad (D, C) \quad \cdots \\
   0 \quad 3 \quad 0 \quad 3 \quad \cdots
   \]

   whose normalized discounted sum is \(3\delta/(1 + \delta)\).

   One-shot deviation (OSD) yields
   
   \[
   (D, D) \quad (D, D) \quad (D, D) \quad (D, D) \quad \cdots \\
   1 \quad 1 \quad 1 \quad 1 \quad \cdots
   \]

   whose normalized discounted sum is 1. Optimality requires

   \[
   3\delta/(1 + \delta) \geq 1 \iff \delta \geq \frac{1}{2}
   \]
iv. **Histories that end with** $(D,D)$

Playing according to the strategy yields:

$$
\begin{array}{cccc}
(D,D) & (D,D) & (D,D) & (D,D) \\
1 & 1 & 1 & 1 \\
\end{array} \cdots
$$

whose normalized discounted sum is 1.

One-shot deviation (OSD) yields

$$
\begin{array}{cccc}
(C,D) & (D,C) & (C,D) & (D,C) \\
0 & 3 & 0 & 3 \\
\end{array} \cdots
$$

whose normalized discounted sum is $3\delta/(1+\delta)$. Optimality requires

$$3\delta/(1+\delta) \leq 1 \leftrightarrow \delta \leq \frac{1}{2}$$

Therefore, this strategy profile is a SPE if and only if $\delta = 1/2$.

(b) **Pavlov**: There are three type of histories: (i) Those that end with $(C,C)$; (ii) Those that end with $(D,D)$; (iii) All other histories.

i. **Histories that end with** $(C,C)$

Playing according to the strategy yields:

$$
\begin{array}{cccc}
(C,C) & (C,C) & (C,C) & (C,C) \\
2 & 2 & 2 & 2 \\
\end{array} \cdots
$$

whose normalized discounted sum is 2.

One-shot deviation (OSD) yields

$$
\begin{array}{cccc}
(D,C) & (C,D) & (C,C) & (C,C) \\
3 & 1 & 2 & 2 \\
\end{array} \cdots
$$

whose normalized discounted sum is

$$(1-\delta)(3 + \delta + \frac{2\delta^2}{1-\delta}) = 3 - 2\delta + \delta^2$$

Optimality requires

$$3 - 2\delta + \delta^2 \leq 2 \leftrightarrow (\delta - 1)^2 \leq 0$$

which is impossible. Therefore, this strategy profile is not a SPE.

2. (a) **Tit-For-Tat**:

i. **Histories that end with** $(C,C)$

Playing according to the strategy yields:

$$
\begin{array}{cccc}
(C,C) & (C,C) & (C,C) & (C,C) \\
4 & 4 & 4 & 4 \\
\end{array} \cdots
$$

whose normalized discounted sum is 4.

One-shot deviation (OSD) yields

$$
\begin{array}{cccc}
(D,C) & (C,D) & (D,C) & (C,D) \\
6 & 0 & 6 & 0 \\
\end{array} \cdots
$$

whose normalized discounted sum is $6/(1+\delta)$. Optimality requires

$$4 \geq \frac{6}{1+\delta}$$

or

$$\delta \geq \frac{1}{2}$$
ii. **Histories that end with** \((C, D)\)  
Playing according to the strategy yields:
\[
(D, C) \quad (C, D) \quad (D, C) \quad (C, D) \quad \cdots
\]
whose normalized discounted sum is \(6/(1 + \delta)\).
One-shot deviation (OSD) yields
\[
(C, C) \quad (C, C) \quad (C, C) \quad (C, C) \quad \cdots
\]
whose normalized discounted sum is 4. Optimality requires
\[
\delta \leq \frac{1}{2}
\]

iii. **Histories that end with** \((D, C)\)  
Playing according to the strategy yields:
\[
(C, D) \quad (D, C) \quad (C, D) \quad (D, C) \quad \cdots
\]
whose normalized discounted sum is \(6\delta/(1 + \delta)\).
One-shot deviation (OSD) yields
\[
(D, D) \quad (D, D) \quad (D, D) \quad (D, D) \quad \cdots
\]
whose normalized discounted sum is 1. Optimality requires
\[
6\delta/(1 + \delta) \geq 1 \Leftrightarrow \delta \geq \frac{1}{5}
\]

iv. **Histories that end with** \((D, D)\)  
Playing according to the strategy yields:
\[
(D, D) \quad (D, D) \quad (D, D) \quad (D, D) \quad \cdots
\]
whose normalized discounted sum is 1.
One-shot deviation (OSD) yields
\[
(C, D) \quad (D, C) \quad (C, D) \quad (D, C) \quad \cdots
\]
whose normalized discounted sum is \(6\delta/(1 + \delta)\). Optimality requires
\[
6\delta/(1 + \delta) \leq 1 \Leftrightarrow \delta \leq \frac{1}{5}
\]
Therefore, there is no \(\delta\) that satisfies all these conditions and hence this strategy profile is not a SPE.

(b) **Pavlov**: There are three type of histories: (i) Those that end with \((C, C)\); (ii) Those that end with \((D, D)\); (iii) All other histories.

i. **Histories that end with** \((C, C)\)  
Playing according to the strategy yields:
\[
(C, C) \quad (C, C) \quad (C, C) \quad (C, C) \quad \cdots
\]

One-shot deviation (OSD) yields
\[
(D, C) \quad (D, D) \quad (C, C) \quad (C, C) \quad \cdots
\]
Payoffs are the same starting with third period. Therefore, optimality requires \(4 + 4\delta \geq 6 + \delta\), or \(\delta \geq 2/3\).
ii. Histories that end with \((D, D)\)

Playing according to the strategy yields:

\[
\begin{array}{cccccc}
(C, C) & (C, C) & (C, C) & (C, C) & \cdots \\
4 & 4 & 4 & 4 & \cdots 
\end{array}
\]

One-shot deviation (OSD) yields

\[
\begin{array}{cccccc}
(D, C) & (D, D) & (C, C) & (C, C) & \cdots \\
6 & 1 & 4 & 4 & \cdots 
\end{array}
\]

Payoffs are the same starting with third period. Therefore, optimality requires \(4 + 4\delta \geq 6 + \delta\), or \(\delta \geq 2/3\).

iii. All other histories

Playing according to the strategy yields:

\[
\begin{array}{cccccc}
(D, D) & (C, C) & (C, C) & (C, C) & \cdots \\
1 & 4 & 4 & 4 & \cdots 
\end{array}
\]

One-shot deviation (OSD) yields

\[
\begin{array}{cccccc}
(C, D) & (D, D) & (C, C) & (C, C) & \cdots \\
0 & 1 & 4 & 4 & \cdots 
\end{array}
\]

OSD is not profitable after any such history. Therefore, we conclude that this strategy profile is a SPE if and only if \(\delta \geq 2/3\).

3. In the the unique Nash equilibrium of the one-shot Cournot game firms produce

\[Q^c = \frac{a - c}{3}\]

and their payoffs are

\[\frac{(a - c)^2}{9}\]

Half the monopoly output is

\[Q^m = \frac{a - c}{4}\]

with payoff

\[\frac{(a - c)^2}{8}\]

There are two categories of histories that are relevant for the grim-trigger strategy:

(a) Histories in which both firms have always produced half the monopoly output

Grim-trigger leads to average discounted payoff of

\[\frac{(a - c)^2}{8}\]

We have to check against all possible one-shot deviations. However, checking against the best OSD is enough. The best deviation is the output level \(Q_1\) that maximizes

\[(\frac{a - c}{4} - Q_1 - c)Q_1\]

since the other firm is producing \((a - c)/4\) and after a deviation each firm is going to produce \(Q^c\) forever. This maximizer is given by

\[Q^d = \frac{3(a - c)}{8}\]

with corresponding one-period payoff of

\[\frac{(a - c)}{8} - \frac{3(a - c)}{8} - c\] \[\frac{3(a - c)}{8} = \frac{9(a - c)^2}{64}\]

Corresponding average discounted payoff is

\[(1 - \delta)(\frac{9(a - c)^2}{64} + \frac{\delta(a - c)^2}{9} + \frac{\delta^2(a - c)^2}{9} + \cdots) = (1 - \delta)(a - c)^2\frac{9}{64} + \frac{\delta}{9(1 - \delta)}\]

Therefore, we need

\[\frac{(a - c)^2}{8} \geq (1 - \delta)(a - c)^2\frac{9}{64} + \frac{\delta}{9(1 - \delta)} \Leftrightarrow \delta \geq \frac{9}{17}\]
(b) All other histories

After such histories, firm 2 always produces the Cournot output, to which producing the Cournot output every period is a best response.

Therefore, this strategy profile is a SPE if and only if $\delta \geq \frac{9}{17}$. 