1. Consider the Cournot duopoly game introduced in the class (see page 6 of Lecture Notes 1) with linear demands and constant unit costs with the exception that each firm has a different unit cost, i.e. the cost functions are given by $C_i(Q_i) = c_i Q_i$, with $a > c_1 > c_2$.

(a) Which firm produces more in a Nash equilibrium of this game?
(b) What is the effect of technical change that lowers firm 2’s unit cost $c_2$ (while not affecting firm 1’s unit cost) on the firms’ equilibrium outputs, the total output, and the price?

2. Consider a variant of the Cournot duopoly game which differs from the one introduced in the class only in that each firm cares about sales, i.e., $Q_i$, as well as profits. In particular, we assume that firm $i$ has the following payoff function

$$u_i(Q_1, Q_2) = (1 - \alpha_i) \text{ (profits of } i) + \alpha_i \text{ (sales of } i),$$

where $0 \leq \alpha_i \leq 1$. You may consider $\alpha_i$ as a measure of firm $i$’s desire to capture a high market share. Set $a = 2$, $b = 1$, $c = 1$, and find the Nash equilibria of this game, and interpret your results when

(a) $\alpha_1 = \alpha_2 > 0$;
(b) $\alpha_1 > \alpha_2$.

(Be careful to include the capacity constraints into your calculations, i.e., neither firm can produce more than $a/b = 2$).

3. Two individuals are involved in a synergistic relationship. If both individuals devote more effort to the relationship, they are both better off. For any given effort of individual $j$, the return to individual $i$’s effort first increases, then decreases. Specifically, an effort level is a nonnegative number, and individual $i$’s preferences (for $i = 1, 2$) are represented by the payoff function $e_i (c + e_j - e_i)$, where $e_i$ is $i$’s effort level, $e_j$ is the other individual’s effort level and $c > 0$ is a constant. Formulate the strategic form of this game and find its Nash equilibria.
4. Two neighbors are planning to clean their street on a Sunday. They each have an hour to spend and they can spend it either by watching TV or by cleaning the street. So, if we denote the amount of time contributed by person \(i\) by \(c_i\), where \(0 \leq c_i \leq 1\), that person spends her remaining time \(1 - c_i\) by watching TV. Each person cares about both how clean the street is and the amount of time they get to watch TV. In particular, we assume that the payoff function of individual \(i\) \((i = 1, 2)\) is given by

\[
u_i(c_1, c_2) = b_i \ln (c_1 + c_2) + 1 - c_i,
\]

where \(b_i \ln (c_1 + c_2)\) represents the utility that the individual \(i\) derives from having a clean street, and \(1 - c_i\) represents the utility of watching TV. You can interpret \(b_i\) as the relative value individual \(i\) attaches to having a clean street.

(a) Write down the strategic form of this game.

(b) Find the Nash equilibria of this game when

i. \(b_1 > b_2\)

ii. \(b_1 = b_2\).