1. **(Centipede Game)** Find the backward induction equilibrium of the game in the following figure.

![Figure 1: A six-period centipede game.](image)

2. A child’s action $a$ affects both her own private income $c(a)$ and her parent’s income $p(a)$; for all values of $a$ we have $c(a) < p(a)$. The child is selfish: she cares only about the amount of money she has. The parent cares both about how much money she has and how much her child has. Specifically, her payoff is the smaller of the amount of money she has and the amount of money her child has. The parent may transfer money to her child. First the child takes an action, and after observing the action the parent decides how much money to transfer to the child. Show that in a subgame perfect equilibrium of this game the child takes an action that maximizes the sum of her private income and her parent’s income.

3. Consider the ultimatum bargaining game as introduced in the class with the following modification. If the share of player $i$ is $x_i$ and that of player $j$ is $x_j$, where $j \neq i$, then the payoff of player $i$ is

$$x_i - \beta x_j$$

where $\beta > 0$. The parameter $\beta$ can be interpreted as a measure of envy of player $i$ towards player $j$. Find the subgame perfect equilibria of this game.

4. Consider the following bargaining game in which two players are trying to share a cake of size 1. Player 1 offers $x_1 \in [0, 1]$ and player 2 either accepts ($Y$) or rejects ($N$). If player 2 accepts player 1 receives a payoff of $x_1$ and player 2 receives $1 - x_1$. If player 2 rejects, then player 2 moves again to offer $x_2 \in [0, 1]$ to which player 1 responds by either accepting ($Y$) or rejecting ($N$). If player 1 accepts player 1 and 2’s payoffs are $\delta (1 - x_2)$ and $\delta x_2$ respectively, where $\delta \in (0, 1)$ is the common discount factor for the
players. If player 1 rejects the offer then an arbitrator terminates the bargaining process and gives player 1 a share $y$ and player 2 the rest which, because of discounting, players value as $\delta^2 y$ and $\delta^2 (1 - y)$. Find the SPE of this game.