KOÇ UNIVERSITY
MATH 101 - FINITE MATHEMATICS
Midterm I          October, 2015
Duration of Exam: 100 minutes

INSTRUCTIONS: You can use calculators in the exam. No books, no notes, and no talking allowed. You must always explain your answers and show your work to receive full credit. Use the back of these pages if necessary. Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.

Name: ___________________________
Surname: _________________________
Signature: _________________________

Section (Check One):
Section 1: Mine Çağlar M-W (8:30)
Section 2: Mine Çağlar M-W (10:00)
Section 3: Ayberk Zeytin Tu-Th (13:00)
Section 4: Ayberk Zeytin Tu-Th (16:00)

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>POINTS</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
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<tr>
<td>2</td>
<td>25</td>
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<td>3</td>
<td>22</td>
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<td>4</td>
<td>23</td>
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<td>5</td>
<td>15</td>
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<tr>
<td>TOTAL</td>
<td>105</td>
<td></td>
</tr>
</tbody>
</table>

A list of formulas: \( I = Prt; A = P(1 + rt) \)

\[
A = P(1+i)^n; \quad APY = (1 + \frac{r}{m})^m - 1; \quad A = Pe^t; \quad APY = e^r - 1;
\]

\[
FV = PMT \frac{(1+i)^n-1}{i}; \quad PV = PMT \frac{1-(1+i)^{-n}}{i}, \text{ where } i = \frac{r}{m} \text{ and } n = mt
\]
1. (a) (12 points) Solve the following equation for \( x \)

\[
2 \log_b(x - 3) - \log_b(2x - 14) = \log_b x
\]

where \( b > 0, b \neq 1 \).

\[
\Rightarrow \log_b \left( \frac{(x-3)^2}{2x-14} \right) = \log_b x
\]

\[
\Rightarrow \frac{(x-3)^2}{2x-14} = x
\]

\[
(x-3)^2 = 2x - 14
\]

\[
x^2 - 6x + 9 = 2x^2 - 14x
\]

\[
0 = x^2 - 8x - 9 = (x+1)(x-9)
\]

\[
x = -1 \text{ or } x = 9
\]

(b)(8 points) Find \( x \) if \( (x - 1)^{202} = (x + 1)^{101} \).

\[
\Rightarrow \left( \frac{(x-1)^2}{x+1} \right)^{101} = (x+1)^{101}
\]

\[
\Rightarrow (x-1)^2 = x+1
\]

\[
x^2 - 2x + 1 = x + 1
\]

\[
x^2 - 3x = 0
\]

\[
x(x-3) = 0
\]

\[
x = 0 \text{ or } x = 3
\]
2. (25 points) To expand your business you need a loan of 8000 TL. Your banker loans you the money at 12% compounded monthly which you agree to repay in 4 equal monthly payments.
(a) (7 points) Find your monthly payments.

\[ 8000 = \frac{PMT}{1 - \left(1 + \frac{0.12}{12}\right)^{-4}} \]

\[ \Rightarrow PMT = 2050.25 \text{ TL} \]

(b) (18 points) In the following amortization schedule, find the numbers A, B and C.

<table>
<thead>
<tr>
<th>Payment Number</th>
<th>Payment</th>
<th>Interest</th>
<th>Unpaid Balance Reduction</th>
<th>Unpaid Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>8000</td>
</tr>
<tr>
<td>1</td>
<td>2050.25</td>
<td>A</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2050.25</td>
<td>***</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2050.25</td>
<td></td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2050.25</td>
<td></td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

\[ A = 8000 \left(\frac{0.12}{12}\right) = 80 \text{ TL} \]
\[ \Rightarrow * = 2050.25 - 80 = 1970.25 \text{ TL} \]
\[ \Rightarrow *** = 8000 - 1970.25 = 6029.75 \text{ TL} \]
\[ \Rightarrow *** = 6029.75 \left(0.01\right) = 60.30 \text{ TL} \]
\[ \Rightarrow B = 2050.25 - 60.30 = 1989.95 \text{ TL} \]
\[ C = \frac{2050.25}{1 - (1 + 0.01)^{-1}} \]
\[ \Rightarrow C = 2029.35 \text{ TL} \]
3. (22 points) A person makes payments of 1500 TL at the end of each quarter into a savings account paying 6.5% compounded quarterly for 5 years.

(a) (12 points) How much is in the account at the end of 5 years?

\[
FV = 1500 \left(1 + \frac{0.065}{4}\right)^{5 \times 4} - 1
\]

\[
= 35115.67
\]

(b) (10 points) Instead, if the person deposited 30,000 TL now into another account compounded continuously, what would be the interest rate in order to receive the same amount as in part (a) at the end of 5 years? Express your answer as percentage, rounded to three decimal places.

\[
A = 30000 e^{r \times 5} = 35115.67
\]

\[
e^{5r} = \frac{35115.67}{30000}
\]

\[
l^{5r} = \ln\left(\frac{35115.67}{30000}\right)
\]

\[
\Rightarrow r \approx 0.03149
\]

\[
\Rightarrow r = 3.149 \%
\]
4. (23 points) Suppose you take out a loan of 200,000 TL mortgage for 10 years at an annual interest rate of 12% which you agree to repay in equal monthly payments.

(a) (7 points) Find your monthly payments.

\[
200000 = \text{PMT} \frac{1 - (1 + 0.12)^{-120}}{0.01}
\]

\[
\Rightarrow \text{PMT} = 2869.42 \text{ TL}
\]

(b) (8 points) Find your unpaid balance at the end of 3 years.

\[
\text{PV} = 2869.42 \frac{1 - (1 + 0.01)^{-7 \times 12}}{0.01}
\]

\[
\approx 162548.20 \text{ TL}
\]

(c) (8 points) How much will you save in interest if you agree to refinance your remaining 7 years of debt with 8.4% annual interest rate, compounded monthly?

\[
162548.20 = \text{PMT} \frac{1 - (1 + 0.0084)^{-84}}{0.0084}
\]

\[
\Rightarrow \text{PMT} = 2566.03 \text{ TL}
\]

In each payment, we save \(2869.42 - 2566.03 = 303.39 \text{ TL}\)

Total interest saved: \(84 \times 303.39 = 25484.76 \text{ TL}\)
5. (15 points) Consider the following system of linear equations,

\[
\begin{align*}
2x + ty &= 6 \\
x + y &= 2
\end{align*}
\]

where \( t \) is a real constant.

(a) (5 points) Find all \( t \) for which the system has no solution.

\[
\begin{align*}
2x + ty &= 6 \\
x + y &= 2
\end{align*}
\]

\[
\Rightarrow \quad t \left( -2 \right) y = -4 \quad (\star)
\]

If \( t \neq 2 \), then

the system has no solution for \((x, y)\)

(b) (6 points) Find all \( t \) for which the above system has a unique solution.

If \( t \neq 2 \), then

\[
y = \frac{2}{t-2}
\]

from \((\star)\)

and since \( x + y = 2 \), we get \( x = 2 - \frac{2}{t-2} \).

Therefore, for \( t \in \mathbb{R} \setminus \{2\} \), the system has a unique solution given by \((\frac{2}{t-2}, \frac{2}{t-2})\).

(c) (4 points) Does there exist \( t \) for which the system has infinitely many solutions? Explain.

There are no such values of \( t \) because one cannot reduce one equation to \( 0 = 0 \) form.