KOÇ UNIVERSITY
MATH 101 - FINITE MATHEMATICS
Final Exam January 7, 2015
Duration of Exam: 90 minutes

INSTRUCTIONS: Calculators may be used on the test. No cell phones, no books, no notes, and no talking are allowed. You must always explain your answers and show your work to receive full credit. Use the back of these pages if necessary. Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.

Name: ________________________________

Surname: ______________________________

Signature: ______________________________

Section (Check One):

Section 1: Selda Küçükcifçi M-W (8:30) ———
Section 2: Selda Küçükcifçi M-W (10:00) ———
Section 3: Haluk Oral M-W(13:00) ———
Section 4: Haluk Oral M-W(16:00) ———
Section 5: Ayberk Zeytin T-Th(8:30) ———

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>POINTS</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>105</td>
<td></td>
</tr>
</tbody>
</table>
1. (a) (10 points) Solve the following equation for $x$;

\[
\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4
\]

(b) (10 points) If $\sin \theta = a$, then write $\sin 3\theta$ in terms of $a$. 
2. (a) (11 points) Let \( A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 1 \\ 2 & 2 & -1 \end{bmatrix}, \ B = \begin{bmatrix} 2 & 0 & -1 \\ -2 & 1 & 2 \end{bmatrix} \) and \( C = \begin{bmatrix} 3 & -3 \end{bmatrix} \).

Determine the followings, if they exist;

(i) \( ABC \)

(ii) \( CBA \)

(b) (9 points) Use Gauss Jordan elimination to bring the following augmented matrix into their reduced form. Write the solution set for the corresponding system.

\[
\begin{bmatrix}
1 & 0 & 1 & 2 \\
0 & 1 & 1 & 3 \\
0 & -2 & -2 & 6 \\
1 & 1 & 2 & 5
\end{bmatrix}
\]
3. (20 points) Using the **simplex method** maximize the objective function \( P = x_1 + 2x_2 \)
subject to the constraints
\[
\begin{align*}
-x_1 + x_2 & \leq 2 \\
-x_1 + 3x_2 & \leq 12 \\
x_1 - 4x_2 & \leq 4 \\
x_1, x_2 & \geq 0 .
\end{align*}
\]
4. (9 points) WRITE THE DECISION VARIABLES, APPROPRIATE EQUATION(S) AND INEQUALITIES SO THAT THE PROBLEM BELOW CAN BE SOLVED. DO NOT SOLVE IT!

A farmer owns a 1280 $m^2$ plot of land for growing orange, peach and apple trees. Each orange tree requires 16 $m^2$, each peach tree requires 4 $m^2$, each apple tree requires 8 $m^2$ of land. The farmer employs 4 workers each year whom works 150 hours per year. Each orange tree requires 30 h. of labor, peach tree requires 5 h. of labor and apple tree requires 10 h. of labor every year. Profit per tree is 50 TL, 25 TL and 20 TL for orange, peach and apple tree respectively. Divide the land among the trees to maximize the profit of the farmer.
A list of formulas: \[ I = Prt; \ A = P(1 + rt) \]
\[ A = P(1 + i)^n; \ APY = (1 + \frac{r}{m})^m - 1; \ A = Pe^{rt}; \ APY = e^r - 1; \]
\[ FV = PMT\frac{[(1+i)^n-1]}{i}; \ PV = PMT\frac{[1-(1+i)^{-n}]}{i}, \text{ where } i = \frac{r}{m} \text{ and } n = mt \]

5. (16 points) Gülen establishes an annuity for retirement by depositing $350 for 25 years into an account that pays 12% compounded monthly. Then she plans to make equal monthly withdrawals for 15 years. What will be each monthly withdrawal?
6. (20 points) Ali and Zeynep are buying a house for $250000. They made a 15% down payment and then they signed a 30 year mortgage at 6% annual interest compounded monthly.

(a) Find their monthly payments.

(b) After making 240 payments they decided to pay their remaining debt. What is the amount they have to pay?