INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and talking allowed. You must always explain your answers and show your work to receive full credit. Use the back of these pages if necessary. Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.

Surname, Name: ________________________________

Signature: ________________________________

Section (Check One):

Section 1: S. Küçükçifci
Section 2: T. Albu (9:30)
Section 3: E.Ş. Yazıcı (15:30)
Section 4: T. Albu (12:30)
Section 5: E.Ş. Yazıcı (11:00)

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Problem 1. Calculate the following limits or show that they do not exist.

(a) (5 pts) \( \lim_{x \to 1} \frac{(x - 1)\sqrt{2x}}{|3x - 3|} \)

(b) (4 pts) \( \lim_{x \to 0} \frac{\ln(1 + x)}{\cos x} \)

(c) (6 pts) For which values of \( a \), the function

\[ f(x) = \begin{cases} 
  x^3 - 2x & \text{for } x \leq a \\
  x^2 & \text{for } x > a
\end{cases} \]

is continuous at \( x = a \)?
Problem 2. Find the derivative of the following function $f$ in (a)-(c). Simplify your answers.

(a) (3 pts) $f(x) = e^{x^2+2} + \ln \left( \frac{x + 1}{x^2 + 2} \right)$

(b) (6 pts) $f(x) = (\sin x)^{\cos x}$

(c) (6 pts) $f(x) = \sqrt[3]{\frac{1 + x^3}{1 - x^3}}$

(d) (5 pts) Find the equation of the tangent line at the point $P(1, e)$ to the curve defined by the equation $y = e^{1/x}$. 
Problem 3. Consider the function

\[ f(x) = \frac{e^x}{x + 2} \]

(a) (6 pts) Find the horizontal and vertical asymptotes of the graph of \( f \) if they exist.

(b) (2 pts) Find the intervals on which the function \( f \) is increasing and decreasing.

(c) (2 pts) Determine the local extreme values of the function \( f \).
Problem 4. Calculate the following integrals.

(a) (5 pts) \[ \int \frac{1}{x^2} \sin \frac{1}{x} \, dx \]

(b) (8 pts) \[ \int \frac{3x^2 + 4x + 4}{x^3 + x} \, dx \]

(c) (7 pts) \[ \int e^x \sin x \, dx \]
Problem 5.  (a) (9 pts) The region between the curve \( y = \sin x, \ 0 \leq x \leq \frac{\pi}{4}, \) and the \( x \)-axis is revolved about the \( x \)-axis to generate a solid. Find its volume.

(b) (6 pts) Find the length of the curve \( y = 2x^{3/2} + 1 \) from \( x = 0 \) to \( x = 1. \)
Problem 6. Find the limit of the following sequences \((a_n)\) and determine whether \((a_n)\) converges or diverges.

(a) (4 pts) \(a_n = \frac{(-1)^n \sin(2n + 3)}{3n + 4}\)

(b) (4 pts) \(a_n = \frac{3^n}{n^3}\)
Problem 7. Determine whether the following series converges or diverges. If the series converges, find its sum.

(a) (4 pts) \[ \sum_{n=1}^{\infty} \frac{3n^4 - 2n + 1}{5n^4 - 2n^2 + n} \]

(b) (4 pts) \[ \sum_{n=1}^{\infty} \frac{6}{(2n - 1)(2n + 1)} \]