Question 1 (15 Points):
Find the following limits:

(a) \( \lim_{h \to 0} \frac{\frac{1}{\pi h} - \frac{1}{2}}{h} \)

(b) \( \lim_{x \to 0} \frac{\tan(4x)}{\sin(5x)} \)

(c) Let \( f(x) = \frac{\tan(4x)}{\sin(5x)} \) for \( -\pi/2 < x < \pi/2, \ x \neq 0 \). How would you define \( f(0) \) so that \( f(x) \) is continuous?
Question 2 (15 Points):  
(a) \( y = f(x) \) is a one-to-one function, and the point \((-1, 2)\) is on its graph. Let \( f^{-1}(x) \) be the inverse function of \( f(x) \), and \( f'(x) = \frac{d}{dx} f(x) \) be the derivative of \( f(x) \). The equation of the tangent to \( y = f(x) \) at \((-1, 2)\) is \( y = 2x + b \). Find the following. Justify your answers. 

(i) \( b \)
(ii) \( f^{-1}(2) \)
(iii) \( f'(-1) \)
(iv) \( f^{-1}(f(-1)) \)
(v) \( \frac{d}{dx} f^{-1}(x) \bigg|_{x=2} \)

(b) If \( \sin(x) = -\frac{1}{2} \), then what are all possible values for \( \tan(x) \)?
Question 3 (15 Points):
Let \( f'(x) = \frac{d}{dx} f(x) \) be the derivative of \( f(x) \). Find

(a) \( f'(x) \) for \( f(x) = \sqrt[3]{\sin(x^2)} \)

(b) The slope of the tangent at \((1, -1)\) to the circle \( x^2 + y^2 = 2 \)

(c) The function \( f(x) \) is continuous in the interval \((-5, 3)\). Find all local extrema of \( f(x) \) in the interval \((-5, 3)\) if \( f'(1) \) does not exist and

<table>
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<th>x</th>
<th>((-5, -2))</th>
<th>(-2)</th>
<th>((-2, -1))</th>
<th>(-1)</th>
<th>((-1, 0))</th>
<th>0</th>
<th>((0, 1))</th>
<th>((1, 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) )</td>
<td>-</td>
<td>0</td>
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<td>0</td>
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Question 4 (10 Points):

(a) Find the \( \frac{d}{dx} \int_{\sqrt{x}}^{3x} t^2 \, dt \) using the Fundamental Theorem of Calculus.

(b) Find \( \frac{d}{dx} \int_{\sqrt{x}}^{3x} t^2 \, dt \) by first finding \( \int_{\sqrt{x}}^{3x} t^2 \, dt \), and then taking the derivative of the result.

(c) Find \( \int_{1}^{e} (2(\ln(x) + 1)) \, dx \) given that the derivative of \( x^2 \ln(x) \) is \( 2(\ln(x) + 1) \).
Question 5 (20 Points):
(a) Evaluate
\[ \int_{0.5}^{1} \frac{x^2 + 13}{x^2 + 1} \, dx \]

(b) Find the area between the curve \( y = 2\sqrt{x^2 + 1} \), \( 0 \leq x \leq \sqrt{3} \), and the x-axis
Question 6 (10 Points):

Determine whether the improper integral \( \int_{0}^{\infty} e^{-x} dx \) is convergent or divergent. If the improper integral is convergent, evaluate it.
Question 7 (10 Points):
Determine whether the following sequence is convergent or divergent. If the sequence is convergent, find its limit.

(a) \( a_n = \frac{(-1)^n}{n+1} \)

(b) \( a_n = \frac{\ln(n+1)}{\sqrt{n}} \)
Question 8 (10 Points):
For each of the following series, write the first 2 terms and determine whether the series is convergent or divergent. If the series converges, find its sum.

(a) \( \sum_{n=1}^{\infty} (-1)^n \)

(b) \( \sum_{n=0}^{\infty} \frac{2^{2n}}{3^{n+1}5^n} \)