3. The characteristic equation is $4r^2 - 8r - 5 = 0$, with roots $r = -1/2, 5/2$. The general solution is $y(t) = c_1 e^{-t/2} + c_2 e^{5t/2}$.

6. The characteristic equation is $r^2 - 10r + 25 = 0$, with the double root $r = 5$. The general solution is $y(t) = c_1 e^{5t} + c_2 t e^{5t}$.

**Problem 9** : The characteristic equation is

$$25r^2 - 30r + 9 = 0,$$

with roots $r_1$ and $r_2$.

$$\Delta = 30^2 - 4 \cdot 25 \cdot 9 = 0.$$ 

$$r_1 = r_2 = \frac{30}{2 \cdot 25} = \frac{2}{5}.$$ 

The general solution is $y(t) = c_1 e^{\frac{2}{5}t} + t \cdot c_2 e^{\frac{2}{5}t}$. 
Problem 12: The characteristic equation is

\[ r^2 - 6r + 9 = 0 \]

with roots \( r_1 = r_2 = 3 \).

The general solution is,

\[ y(t) = c_1 e^{3t} + t \cdot c_2 e^{3t} \]

From \( y(0) = 0 \) we get that,

\[ 0 = c_1 \]

So, \( y(t) = c_2 t e^{3t} \).

The derivative is \( y'(t) = c_2 e^{3t} + c_2 t \cdot 3e^{3t} \) and invoking the initial condition \( y'(0) = 3 \) we obtain,

\[ 3 = c_2 e^0 + 0 \Rightarrow c_2 = 3 \]

The solution is \( y(t) = 3te^{3t} \).

As \( t \to \infty \), we have \( y(t) \to \infty \).
Problem 14: the characteristic equation is
\[ r^2 + 4r + 4 = 0 \]
with roots \( r_1 = r_2 = -2 \).

The general solution is
\[ y(t) = c_1 e^{-2t} + c_2 te^{-2t} \]
\[ y'(t) = -2c_1 e^{-2t} + (c_2 e^{-2t} - 2c_2 t e^{-2t}) \]

From \( y(-1) = 2 \) and \( y'(-1) = 3 \) we have
\[
\begin{cases}
2 = c_1 e^2 - c_2 e^2 \\
3 = -2c_1 e^2 + c_2 e^2 + 2c_2 e^2
\end{cases}
\]
So that \( c_2 = e^2 \cdot 4 \) and \( c_1 = 9e^2 \).

Hence,
\[ y(t) = 9e^{-2(t+1)} + 7t e^{-2(t+1)} \]

As \( t \to \infty \) we have (clear) \( y(t) \to 0 \).
16. The characteristic roots are $r_1 = r_2 = 1/2$. Hence the general solution is given by $y(t) = c_1e^{t/2} + c_2te^{t/2}$. Invoking the initial conditions, we require that $c_1 = 2$, and that $1 + c_2 = b$. The specific solution is $y(t) = 2e^{t/2} + (b-1)te^{t/2}$. Since the second term dominates, the long-term solution depends on the sign of the coefficient $b-1$. The critical value is $b = 1$.

23. Set $y_2(t) = t^3 v(t)$. Substitution into the differential equation results in

$$t^2(t^3v'' + 6t^2v' + 6tv) - 4t(t^3v' + 3t^2v) + 6t^3v = 0.$$ 

After collecting terms, we end up with $t^5v'' + 2t^4v' = 0$. Hence $v(t) = c_1 + c_2/t$, and thus $y_2(t) = c_1t^3 + c_2t^2$. Setting $c_1 = 0$ and $c_2 = 1$, we obtain $y_2(t) = t^2$.

25. Set $y_2(t) = t^{-1}v(t)$. Substitution into the differential equation into the differential equation results in

$$t^2(2t^{-3}v - t^{-2}v' + v''t^{-1} - t^{-2}v') + 3t(v't^{-1} - t^{-2}v) + t^{-1}v = 0.$$ 

After collecting terms, we end up with $tv'' + v' = 0$. This equation is linear in variable $w = v'$. It follows that $v'(t) = t^{-1} + c_1$, and $v(t) = \ln(t) + c_1t + c_2$. Thus $y_2(t) = t^{-1}\ln(t) + c_1tt^{-1} + c_2t^{-1} = t^{-1}\ln(t) + c_1 + c_2t^{-1}$. Setting $c_1 = 0$ and $c_2 = 0$, we obtain $y_2(t) = t^{-1}\ln(t)$. 