KOÇ UNIVERSITY
MATH 106 - CALCULUS I
Midterm I November 16, 2011
Duration of Exam: 90 minutes

INSTRUCTIONS: CALCULATORS ARE NOT ALLOWED FOR THIS EXAM. No books, no notes, no questions and no talking allowed. You must always explain your answers and show your work to receive full credit. Use the back of these pages if necessary. Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.

Surname, Name: __________________________________________

Signature: __________________________________________

Section (Check One):

   Section 1: S. Küçükçifçi (Mon-Wed-Fri 12:30) ——
   Section 2: E. Ş. Yazıcı (Mon-Wed-Fri 14:30) ——
   Section 3: S. Küçükçifçi (Mon-Wed-Fri 10:30) ——
   Section 4: E. Ş. Yazıcı (Mon-Wed-Fri 11:30) ——
   Section 5: T. Etgül (Tue-Thu 12:30) ——

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1. Compute the following limits. Specify any infinite limits.

a) (8 points) \( \lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1} = \)

b) (8 points) \( \lim_{x \to 3} \frac{x - 2}{|x - 3|} = \)

c) (8 points) \( \lim_{x \to -\infty} x + \sqrt{x^2 - 4x + 1} = \)
2. (8 points) Find all $A$ which make the following piecewise defined function $f$ continuous for all real numbers

$$f(x) = \begin{cases} 
Ax^2 + 2x & \text{for } x \leq 2 \\
A^2x - 2x + 2 & \text{for } x > 2 
\end{cases}$$

3. (10 points) Show that there is a positive real number which is a solution to the equation $x^4 - 3x^3 + x^2 - 2 = 0$. 
4. a) (6 points) Write the precise definition of \( \lim_{x \to a} f(x) = L \). (\( \epsilon, \delta \) definition of limit)

b) (10 points) Prove that \( \lim_{x \to 0} x \cos(1/x) = 0 \) by using the \( \epsilon, \delta \) definition of limit.
(Hint: \(|\cos(1/x)| \leq 1\))

5. (10 points) Evaluate \( \lim_{\theta \to 0} \frac{e^{\sin \theta} - 1}{\theta} \).
6. Differentiate the following functions.

a) (9 points) \( g(t) = \frac{t}{\cos(t^4)} \)

b) (8 points) \( f(x) = \left(\frac{1}{x}\right)^{\sqrt{x}} \)
7. (10 points) Suppose that $y = 3x - 5$ is tangent to $y = f(x)$ at $(0, -5)$ and; $y = 4x + 2$ is tangent to $y = g(x)$ at $(1, 6)$. Find the slope of the tangent line to the curve given by the equation

$$f(x)g(y) = -30$$

at the point $(0,1)$.

8. (7 points) Show that the derivative of an even differentiable function is an odd function.