HW4

1. (Problem 5.33 in book) Air at 90 kPa and 10°C. enters an adiabatic diffuser steadily with a velocity of 180 m/s and leaves with a low velocity at a pressure of 100 kPa. The exit area of the diffuser is 4 times the inlet area. Determine (a) the exit temperature and (b) the exit velocity.

Assumptions 1 This is a steady-flow process since there is no change with time. 2 Air is an ideal gas with variable specific heats. 3 Potential energy changes are negligible. 4 The device is adiabatic and thus heat transfer is negligible. 5 There are no work interactions.

Properties The enthalpy of air at the inlet temperature of 10°C = 283 K is \( h_1 = 283.14 \text{ kJ/kg} \) (Table A-17).

Analysis (a) There is only one inlet and one exit, and thus \( \dot{m}_1 = \dot{m}_2 = \dot{m} \). We take diffuser as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{\Delta E_{\text{system}}}{\Delta t} = 0
\]

Rate of net energy transfer by heat, work, and mass Rate of change in internal, kinetic, potential, etc energies

\[
\dot{m}(h_1 + \frac{V_1^2}{2}) - \dot{m}(h_2 + \frac{V_2^2}{2}) = 0 \quad \text{(since } \dot{Q} = \dot{W} = \Delta pe \equiv 0 )
\]

or,

\[
h_2 = h_1 - \frac{V_2^2 - V_1^2}{2} = 283.14 \text{ kJ/kg} - \frac{0 - (180 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 299.3 \text{ kJ/kg}
\]

From Table A-17, \( T_2 = 299.1 \text{ K} \)

(b) The exit velocity of air is determined from the conservation of mass relation,

\[
\frac{1}{\nu_2} A_2 V_2 = \frac{1}{\nu_1} A_1 V_1 \rightarrow \frac{1}{RT_2 / \rho_2} A_2 V_2 = \frac{1}{RT_1 / \rho_1} A_1 V_1
\]

Thus,

\[
V_2 = \frac{A_1 \rho_1 P_1}{A_2 \rho_2 P_2} V_1 = \frac{1}{5} \left( \frac{299.1 \text{ K}}{100 \text{ kPa}} \right) (180 \text{ m/s}) = 34.2 \text{ m/s}
\]
2. **(Problem 5.50 in book)** An adiabatic compressor compresses 0.1 m$^3$/s of air at 120 kPa and 20°C to 100 kPa and 300°C. Determine (a) the work required by the compressor, in kJ/kg, and (b) the power required to drive the compressor, in kW.

**Assumptions**
1. This is a steady-flow process since there is no change with time.
2. Kinetic and potential energy changes are negligible.
3. Air is an ideal gas with constant specific heats.

**Properties**
The constant pressure specific heat of air at the average temperature of $(20 + 300)/2 = 160°C = 433 K$ is $c_p = 1.018 \text{ kJ/kg K}$ (Table A-2b). The gas constant of air is $R = 0.287 \text{ kPa·m}^3/\text{kg·K}$ (Table A-1).

**Analysis** (a) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

\[
\dot{E}_{in} - \dot{E}_{out} = \dot{\Delta E}_{system} = 0 \quad \text{(steady)}
\]

\[
\dot{E}_{in} = \dot{E}_{out}
\]

\[
\dot{W}_{in} + \dot{m}(h_1 - h_2) \quad \text{(since \Delta ke = \Delta pe = 0)}
\]

\[
\dot{W}_{in} = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1)
\]

Thus,

\[
\dot{w}_{in} = c_p(T_2 - T_1) - (1.018 \text{ kJ/kg K})(300 - 20)K = 285.0 \text{ kJ/kg}
\]

(b) The specific volume of air at the inlet and the mass flow rate are

\[
\nu_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa·m}^3/\text{kg·K})(20 + 273 K)}{120 \text{ kPa}} = 0.7008 \text{ m}^3/\text{kg}
\]

\[
\dot{m} = \frac{\dot{V}_1}{\nu_1} = \frac{0.010 \text{ m}^3/\text{s}}{0.7008 \text{ m}^3/\text{kg}} = 0.01427 \text{ kg/s}
\]

Then the power input is determined from the energy balance equation to be

\[
\dot{W}_{in} = \dot{m}c_p(T_2 - T_1) = (0.01427 \text{ kg/s})(1.018 \text{ kJ/kg·K})(300 - 20)K = 4.068 \text{ kW}
\]
3. (Problem 5.67 in book) Refrigerant R-134a enters the expansion valve of a refrigeration system at 1200 kPa as a saturated liquid and leaves at 200 kPa. Determine the temperature and internal energy changes across the valve.

**Assumptions**
1. This is a steady-flow process since there is no change with time.
2. Kinetic and potential energy changes are negligible.
3. Heat transfer to or from the fluid is negligible.
4. There are no work interactions involved.

**Analysis**
There is only one inlet and one exit, and thus \( \dot{m}_1 = \dot{m}_2 = \dot{m} \). We take the throttling valve as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \dot{Q} + \dot{W} = \Delta E_{\text{system}} = 0
\]

\[\dot{E}_{\text{in}} = \dot{E}_{\text{out}}\]

\[\dot{m}_1 h_1 = \dot{m}_2 h_2\]

\[h_1 = h_2\]

since \( \dot{Q} = \dot{W} = \Delta ke \neq \Delta pe \neq 0 \). The properties are (Tables A-11 through 13),

\[
\begin{align*}
P_1 &= 1200 \text{ kPa} & h_1 &= 117.77 \text{ kJ/kg} \\
x_1 &= 0 & h_1 &= 116.70 \text{ kJ/kg} \\
T_1 &= 46.29^\circ C & h_1 &= 117.77 \text{ kJ/kg} \\
P_2 &= 200 \text{ kPa} & T_2 &= -10.09^\circ C \\
h_2 &= h_1 & u_2 &= 109.99 \text{ kJ/kg} \\
\Delta T &= T_2 - T_1 &= -10.09 - 46.29 &= -56.4^\circ C \\
\Delta u &= u_2 - u_1 &= 109.99 - 116.70 &= -6.71 \text{ kJ/kg}
\end{align*}
\]

That is, the temperature drops by 56.4°C and internal energy drops by 6.71 kJ/kg.
4. (Problem 5.86 in book) A chilled-water heat-exchange unit is designed to cool 5 m$^3$/s of air at 100 kPa and 30°C to 100 kPa and 18°C by using water at 8°C. Determine the maximum water outlet temperature when the mass flow rate of the water is 2 kg/s. (Answers: 16.3°C).

Assumptions 1 This is a steady-flow process since there is no change with time. 2 Kinetic and potential energy changes are negligible. 3 There are no work interactions. 4 Heat loss from the device to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. 5 Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is 0.287 kPa m$^3$/kg·K (Table A-1). The constant pressure specific heat of air at room temperature is $c_p = 1.005$ kJ/kg·°C (Table A-2a). The specific heat of water is 4.18 kJ/kg·K (Table A-3).

Analysis The water temperature at the heat exchanger exit will be maximum when all the heat released by the air is picked up by the water. First, the inlet specific volume and the mass flow rate of air are

\[ \nu_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(303 \text{ K})}{100 \text{ kPa}} = 0.8696 \text{ m}^3/\text{kg} \]

\[ \dot{m}_a = \frac{\dot{V}_a}{\nu_1} = \frac{5 \text{ m}^3/\text{s}}{0.8696 \text{ m}^3/\text{kg}} = 5.750 \text{ kg/s} \]

We take the entire heat exchanger as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as

**M ass balance (for each fluid stream):**

\[ \dot{m}_\text{in} - \dot{m}_\text{out} = \Delta \dot{m}_\text{system} = 0 \]

**E nergy balance (for the entire heat exchanger):**

\[ E_\text{in} - E_\text{out} = \Delta E_{\text{system}} = 0 \]

Rate of net energy transfer by heat, work, and mass <br> Rate of change in internal, kinetic, potential, etc. energies

\[ \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{m}_4 h_4 \] (since \( \dot{Q} = \dot{W} = \Delta k_e = \Delta p_e = 0 \))

Combining the two,

\[ \dot{m}_a (h_1 - h_2) = \dot{m}_w (h_4 - h_5) \]

\[ \dot{m}_a c_{p,a} (T_1 - T_3) = \dot{m}_w c_{p,w} (T_4 - T_2) \]

Solving for the exit temperature of water,

\[ T_4 = T_2 + \frac{\dot{m}_a c_{p,a} (T_1 - T_3)}{\dot{m}_w c_{p,w}} = 8°C + \frac{(5.750 \text{ kg/s})(1.005 \text{ kJ/kg·°C})(30-18°C)}{(2 \text{ kg/s})(4.18 \text{ kJ/kg·°C})} = 16.3°C \]
5. (Problem 5.99 in book) A computer cooled by a fan contains eight PCBs, each dissipating 10 W power. The height of the PCBs is 12 cm and the length is 18 cm. The cooling air is supplied by a 25-W fan mounted at the inlet. If the temperature rise of air as it flows through the case of the computer is not to exceed 10°C, determine (a) the flow rate of the air that the fan needs to deliver and (b) the fraction of the temperature rise of air that is due to the heat generated by the fan and its motor. (Answers: (a) 0.0104 kg/s, (b) 24%).

Assumptions 1 Steady flow conditions exist. 2 Air is an ideal gas with constant specific heats. 3 The pressure of air is 1 atm. 4 Kinetic and potential energy changes are negligible

Properties The specific heat of air at room temperature is \( c_p = 1.005 \text{ kJ/kg.}^\circ\text{C} \) (Table A-2).

Analysis (a) We take the air space in the computer as the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

\[
\dot{E}_{in} - \dot{E}_{out} = \frac{\Delta E_{system}}{\dot{E}_{system}} (\text{steady}) = 0
\]

\[
\dot{E}_{in} = \dot{E}_{out}
\]

\[
\dot{Q}_{in} + \dot{W}_{in} + n \dot{h}_1 = \dot{m} \dot{h}_2 \quad (\text{since } \Delta ke = \Delta pe = 0)
\]

\[
\dot{Q}_{in} + \dot{W}_{in} = \dot{m}c_p(T_e - T_i)
\]

Noting that the fan power is 25 W and the 8 PCBs transfer a total of 80 W of heat to air, the mass flow rate of air is determined to be

\[
\dot{Q}_{in} + \dot{W}_{in} = \dot{m}c_p(T_e - T_i) \rightarrow \dot{m} = \frac{\dot{Q}_{in} + \dot{W}_{in}}{c_p(T_e - T_i)} = \frac{(8 \times 10) \text{ W} + 25 \text{ W}}{(1005 \text{ J/kg.}^\circ\text{C})(10^\circ\text{C})} = 0.0104 \text{ kg/s}
\]

(b) The fraction of temperature rise of air that is due to the heat generated by the fan and its motor can be determined from

\[
\dot{Q} = \dot{m}c_p \Delta T \rightarrow \Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{25 \text{ W}}{(0.0104 \text{ kg/s})(1005 \text{ J/kg.}^\circ\text{C})} = 2.4^\circ\text{C}
\]

\[
f = \frac{2.4^\circ\text{C}}{10^\circ\text{C}} = 0.24 = 24%}

6. **Problem 5.112 in book** A rigid, insulated tank that is initially evacuated is connected through a valve to a supply line that carries helium at 200 kPa and 120°C. Now the valve is opened, and helium is allowed to flow into the tank until the pressure reaches 200 kPa, at which point the valve is closed. Determine the flow work of the helium in the supply line and the final temperature of the helium in the tank.

*Answers: 816 kJ/kg, 655 K.*

**Properties** The properties of helium are \( R = 2.0769 \text{ kJ/kg.K} \), \( c_p = 5.1926 \text{ kJ/kg.K} \), \( c_v = 3.1156 \text{ kJ/kg.K} \) (Table A-2a).

**Analysis** The flow work is determined from its definition but we first determine the specific volume

\[
\nu = \frac{RT_{\text{line}}}{P} - \frac{(2.0769 \text{ kJ/kg.K})(120 + 273 \text{ K})}{(200 \text{ kPa})} = 4.0811 \text{ m}^3/\text{kg}
\]

\[w_{\text{flow}} = PV = (200 \text{ kPa})(4.0811 \text{ m}^3/\text{kg}) = 816.2 \text{ kJ/kg}\]

Noting that the flow work in the supply line is converted to sensible internal energy in the tank, the final helium temperature in the tank is determined as follows

\[u_{\text{tank}} = h_{\text{line}}\]

\[h_{\text{line}} = c_p T_{\text{line}} = (5.1926 \text{ kJ/kg.K})(120 + 273 \text{ K}) = 2040.7 \text{ kJ/kg}\]

\[u_{\text{tank}} = c_v T_{\text{tank}} \rightarrow 2040.7 \text{ kJ/kg} = (3.1156 \text{ kJ/kg.K})T_{\text{tank}} \rightarrow T_{\text{tank}} = 655.0 \text{ K}\]

**Alternative Solution:** Noting the definition of specific heat ratio, the final temperature in the tank can also be determined from

\[T_{\text{tank}} = \kappa T_{\text{line}} - 1.667(120 + 273 \text{ K}) = 655.1 \text{ K}\]

which is practically the same result.