MATH 205 ALGEBRA I (FALL 2002)
HOMEWORK ASSIGNMENT V

1. Let \( N = \{ a_b : b \in \mathbb{R} \} \). Prove that \( N \) is a normal subgroup of \( G \), where \( G \) is defined as in question 1 of homework assignment III.

2. Let \( G \) be a group, \( H \) a subgroup of \( G \), and \( N \) a normal subgroup of \( G \). Prove that \( H \cap N \) is a normal subgroup of \( H \).

3. a) Let \( G \) be a group, \( H \) a subgroup of \( G \), and \( N \) a normal subgroup of \( G \). Prove that \( NH \) is a subgroup of \( G \).
   b) If both \( N \) and \( H \) are normal subgroups of \( G \), prove that \( NH \) is also a normal subgroup of \( G \).

4. Let \( G \) be a group and \( \{ N_\alpha \} \), \( \alpha \in I \) be a collection of normal subgroups of \( G \). Prove that \( \bigcap_{\alpha \in I} N_\alpha \) is a normal subgroup of \( G \).

5. Let \( G \) be a group and \( N \) and \( M \) be normal subgroups of \( G \) such that \( N \cap M = \{ e \} \). Prove that \( nm = mn \) for all \( n \in N \) and for all \( m \in M \).

6. Let \( G \) be a group, \( H \) a subgroup of \( G \), and \( N \) a normal subgroup of \( G \). Prove that \( HN = NH \).

7. Let \( G \) be a group, \( H \) a subgroup of \( G \), and \( N \) a normal subgroup of \( G \). Prove that \( N \) is a normal subgroup of \( HN \).

8. Let \( p \) and \( q \) be prime numbers. Find the number of generators of the cyclic group \( \mathbb{Z}_pq \).

9. Let \( p \) be a prime number. Find the number of generators of the cyclic group \( \mathbb{Z}_{pr} \), where \( r \) is an integer \( \geq 1 \).

10. Let \( G \) be an abelian group and \( a \) and \( b \) be two elements of finite order of \( G \). Prove that if the orders, \( o(a) \) and \( o(b) \) of \( a \) and \( b \) are relatively prime, then \( o(ab) = o(a)o(b) \).