MATH 205 ALGEBRA I (FALL 2002)  
HOMEWORK ASSIGNMENT VI

1. Prove that the center $H_G$ of a group $G$ as defined in question 4 of HW Assignment IV is a normal subgroup of $G$.

2. If $G$ is a torsion group (i.e. the elements of $G$ are of finite order) and $N$ is a normal subgroup of $G$. Prove that $G/N$ is also a torsion group.

3. Let $G$ and $G'$ be groups and $\phi$ be an homomorphism from $G$ to $G'$. Prove that $\phi(a) = \phi(b)$ if and only if $aKer\phi = bKer\phi$, where $a$ and $b$ are arbitrary elements of $G$.

4. Let $G$ and $G'$ be groups, $A$ and $B$ subgroups of $G$ such that $A$ is a normal subgroup of $B$. Let $\phi$ be an homomorphism from $G$ to $G'$. Prove that $\phi(A)$ is a normal subgroup of $\phi(B)$.

5. Prove that any infinite cyclic group $G$ is isomorphic to the group $(\mathbb{Z}, +)$.

6. Prove that any two cyclic groups of the same order is isomorphic.

7. Let $G$ be an abelian group. Prove that if the group $G'$ is isomorphic to $G$ then $G'$ is abelian also.

8. Let $G$ be a cyclic group. Prove that if the group $G'$ is isomorphic to $G$ then $G'$ is cyclic also.

9. Let $G$ be the group defined as in question 1 of HW Assignment III and $N$ be a normal subgroup defined as in question 1 of HW Assignment V. Prove that $G/N$ is isomorphic to $(\mathbb{R}^*, \cdot)$. 