Math 302. Homework 7

Solve the following problems:

(1) Show that if \( f \in L^1(a, b) \) then \(|f| \in L^1(a, b)\) and

\[
\left| \int_a^b f(x) \, dx \right| \leq \int_a^b |f(x)| \, dx.
\]

(2) Show that, if \( f, g \in L^1(a, b) \) and \( f \leq g \) a.e. in \((a, b)\), then \( \int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx \).

(3) Show that, if \( f, g \in L^1(a, b) \) then \( \max(f(x), g(x)), \min(f(x), g(x)) \in L^1(a, b) \).

(4) Show that if \( f \in L^1(a, b) \) and \( \epsilon > 0 \), then there exists a step function \( \phi \) such that

\[
\int_a^b |f(x) - \phi(x)| \, dx < \epsilon
\]

(5) True or false? Why?

a) the function \( f(x) = \frac{1}{x(1-x)} \) is measurable on \((0, 1)\),

b) the function \( f(x) = \frac{1}{x} \) is Lebesgue integrable over \((0, 1)\).

(6) Given an example of a function which is Lebesgue integrable over \((0, 1)\), but is unbounded on each interval \((a, b) \subset (0, 1)\).

(7) Show that if a function \( f \) is differentiable on \([0, 1]\), then the function \( f' \) is measurable.