Math 302. Homework 7a

Solve the following problems:

(1) Show that $L^2(0, 1)$ is a separable Hilbert space.

(2) Show that if $E, F \subset [0, 1]$ are measurable sets, then are measurable, then the sets

$[a, b] \setminus E, E \cap F, E \cap F, E \setminus F$

are also measurable .

(3) Show that $f$ is integrable on $(0, 1)$ if and only if $|f|$ integrable on $(0, 1)$.

(4) Find the norms of the function $f(x) = x^\alpha$ in $L^p(0, 1), p > 1$, for those values of $\alpha$

for which $f \in L^p(0, 1)$.

(5) Show that if $1 \leq p < q$, then and $f \in L^q(0, 1)$, then $f \in L^p(0, 1)$ and

$\|f\|_{L^p} \leq \|f\|_{L^q}$. 