Comp 106, HW1, Fall 2017

(The problems below are all from the first chapter of the 6th edition of your textbook. The same problems (or very similar ones) are also available in the 7th edition).

Please provide formal justification for all your answers to the following questions in order to get full credit.

**Question 1**

Is the assertion “This statement is false” a proposition?

**Question 2**

Are these system specifications consistent? “If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer.”

**Question 3**

Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent.

**Question 4**

Express the negation of these propositions using quantifiers, and then express the negation in English.

a) Some drivers do not obey the speed limit.

b) All Swedish movies are serious.

c) No one can keep a secret.

d) There is someone in this class who does not have a good attitude.
In Question 4, use the following predicates (or propositional functions):

(a) $P(x)$: “$x$ obeys the speed limit”
   $Q(x)$: “$x$ is a driver”
The universe of discourse is the set of all people.

(b) $P(x)$: “$x$ is serious”
   $Q(x)$: “$x$ is Swedish”
The universe of discourse is the set of all movies.

(c) $P(x)$: “$x$ can keep a secret”
The universe of discourse is the set of all people.

(d) $P(x)$: “$x$ is in this class”
   $Q(x)$: “$x$ has a good attitude”
The universe of discourse is the set of all students.

**Question 5**

Determine whether $\forall x (P(x) \leftrightarrow Q(x))$ and $\forall x \ P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

**Question 6**

Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements “$x$ is a duck,” “$x$ is one of my poultry,” “$x$ is an officer,” and “$x$ is willing to waltz,” respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

a) No ducks are willing to waltz.
b) No officers ever decline to waltz.
c) All my poultry are ducks.
d) My poultry are not officers.
e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?
Question 7

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the words “It is not the case that.”)

a) No one has lost more than one thousand dollars playing the lottery.
b) There is a student in this class who has chatted with exactly one other student.
c) No student in this class has sent e-mail to exactly two other students in this class.
d) Some student has solved every exercise in this book.
e) No student has solved at least one exercise in every section of this book.

In Question 7, use the following predicates (or propositional functions):

(a) $Q(x,y)$: "Person $x$ has lost $y$ dollars playing the lottery"
The universe of discourse for $x$ is the set of all people.
The universe of discourse for $y$ is the set of all numbers.

(b) $Q(x,y)$: "Student $x$ has chatted with student $y$"
The universe of discourse for $x$ and $y$ is the set of students in this class.

(c) $Q(x,y)$: "Student $x$ has sent email to student $y$"
The universe of discourse for $x$ and $y$ is the set of students in this class.

(d) $Q(x,y)$: “Student $x$ solved exercise $y$”  
$P(y)$: “Exercise $y$ is in the book”
The universe of discourse for $x$ is the set of all students.  
The universe of discourse for $y$ is the set of all exercises.

(e) $R(x,y,z)$: “Student $x$ has solved exercise $y$ in section $z$ of the book”

The universe of discourse for $x$ is the set of all students.  
The universe of discourse for $y$ is the set of exercises in the book.  
The universe of discourse for $z$ is the set of sections in the book.
Question 8 (answer only parts (a) and (c))

For each of these arguments determine whether the argument is correct or incorrect and explain why.

a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

b) A convertible car is fun to drive. Isaac’s car is not a convertible. Therefore, Isaac’s car is not fun to drive.

c) Quincy likes all action movies. Quincy likes the movie *Eight Men Out*. Therefore, *Eight Men Out* is an action movie.

Question 9

Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

Question 10

Prove that $\sqrt{2}$ is irrational.

Question 11

Prove that if $x^3$ is irrational, then $x$ is irrational.