The problems below are all very similar to or the same as the questions in the Algorithms chapter of your textbook.

Please provide formal justification for all your answers to the following questions in order to get full credit.

1. The **selection sort** begins by finding the least element in the list. This element is moved to the front. Then the least among the remaining elements is found and put into the second position. This procedure is repeated until the entire list has been sorted. Write the selection sort algorithm in pseudocode. (For the pseudocode, you can use the textbook’s notation or the one that we used in the class.)

2. Show that $x^5$ is **not** $O(x^2)$ but that $x^2$ is $O(x^5)$.

3. Use the definition of big-$O$ notation “$f(x)$ is $O(g(x))$” to show that $4^x + 65$ is **not** $O(2^x)$.

4. Show that if $f(x)$ is $O(5x^2)$ then $f(x)$ is also $O(4x^4)$.

5. Give a big-$O$ estimate for $f(n) = (n2^n + 3 \cdot 5^n)(\log n! + 5^n)$. For the function $g$ in your estimate “$f(x)$ is $O(g(x))$”, use a simple function $g$ of the smallest possible order.

6. Let $f(x) = 5\sqrt{x\log x}$. Is $f(x)$ $\Omega(x)$? Is $f(x)$ $O(x)$? Explain your answers.

7. Show that $2x^3 + x - 7$ is $\Theta(x^3)$.

8. Show that $\log_2 x$ is $\Theta(\log_{10} x)$.

9. Suppose that $f(x)$ is $O(g(x))$. Does it follow that $3f(x)$ is $O(3g(x))$?

10. Suppose that $f(x)$, $g(x)$, and $h(x)$ are functions such that $f(x)$ is $\Theta(g(x))$ and $g(x)$ is $\Theta(h(x))$. Show that $f(x)$ is $\Theta(h(x))$.

11. Write down the complexity function $f(n)$ of the selection sort algorithm (see Question 1) to sort $n$ items, in terms of **comparison** and **arithmetic** operations. Use your answer to give a big-$O$ estimate and a big-$\Theta$ estimate of the complexity of the selection sort algorithm. Note that whether or not you count also assignment operations while computing the complexity function does not effect your estimates.